MICROWAVE TRANSMISSION LINES

Introduction :-

The electromagnetic waves ranging from 1GHz to 300GHz are generally known as microwaves. Signals at these frequencies have wavelengths ranging from 30cm (at 1GHz) to 1mm (at 300GHz) ie, the wavelengths of the microwaves is very short hence these are called microwaves.

Microwave Spectrum & Bands :-

→ Position of microwave bands in the electromagnetic spectrum.

Band Designation	Frequency Range
Ultra Low Frequencies (ULF)	(3 - 30) Hz
Extreme Low Frequencies (ELF)	(30 - 300) Hz
Voice Frequency (VF) Telephone / Base band frequencies	(300 - 3000) Hz
Very Low Frequencies (VLF)	(3 - 30) KHz
Low Frequency (LF)	(30 - 300) KHz
Medium Frequency (MF)	(300 - 3000) KHz
High Frequency (HF)	(3 - 30) MHz
Very High Frequency (VHF)	(30 - 300) MHz
Ultra High Frequency (UHF)	(300 - 3000) MHz (or) (0.3 - 3) GHz
Super High Frequency (SHF)	(3 - 30) GHz
Extreme High Frequency (EHF)	(30 - 300) GHz
Submillimetre	(300 - 3000) GHz

UHF, SHF, EHF: Microwave frequencies

→ IEEE (Institute of Electrical & Electronics Engineers) Microwave frequency Bands.

Band Designation	Frequency Range (GHz).
UHF	0.3 – 1
L Band	1 – 2
S Band	2 – 4
C Band	4 – 8
X Band	8 – 12
Ku Band	12 – 18
K Band	18 – 27
Ka Band	27 – 40
Millimeter	40 – 300
Submillimeter	>300

Advantages of Microwaves :-

1. Increased Bandwidth availability :-

Microwaves have large bandwidths due to higher frequencies. The advantage of large bandwidth is that The frequency range of information channels will be small percentage of the carrier frequency and more information can be transmitted in microwave frequency ranges. Microwave region is very useful since the lower band of frequency is already crowded.

2. Improved directive Properties :-

As frequency increases, directivity increases and beam width decreases. Hence the beam width of radiation $\theta \propto \lambda/D$.

It is very difficult to produce sharp beams of radiation at low frequency bands, due to large diameter of the lens. However at microwave frequencies, production of extremely [illegible] because of the small diameter of the antenna. Similarly high gain is also achievable at microwave frequencies but which is just impossible at lower frequency bands.

$$G = \frac{4\pi}{\lambda^2} A.$$
$$G \propto 1/\lambda^2 \Rightarrow G \propto f^2.$$

3. **Fading Effect & Reliability :-**

Fading effect due to variation in the transmission medium is more effective at low frequency. Due to "Line of Sight" (LOS) propogation & high frequencies there is less fading effect & hence microwave communication is more reliable.

4. **Power Requirements :-**

Transmitter | Receiver Power requirements are very low at microwave frequencies compared to that at low frequency band

5. **Transparency Property of Microwaves :-**

Microwave frequency band ranging from 300 MHz - 10 GHz are capable of freely propogating through the ionized layers surround the earth as well as through the atmosphere. The presence of such a transparent window in microwave band facilitates the study of microwave radiation from the sun & stars in radio astronomical research of space. It also makes it possible for duplex communication and exchange of information between ground stations and space vehicles.

Applications of Microwave Frequencies :-

Microwaves have a broad range of applications as listed below.

1. long distance communications
2. Radio astronomical research of space
3. Commercial & Industrial applications
4. Radars
5. Remote Sensing
6. Electronic warfare
7. other applications

1. Long Distance Communications :-

Microwave frequencies are used in communication links such as telephone, T.V, telemetry communications, satellite & space. In TV & Radio broadcast applications microwaves are used as the carrier signal for audio & video signals.

2. Radio Astronomical Research of space :-

Microwave receivers are used in radio astronomy to study and detect the electromagnetic radiations from the sun & the stars. These receivers are also used to detect the noise radiated from plasmas.

3. Commercial & Industrial applications :-

Commercial & industrial applications use heat property of microwaves.

(i) Microwave oven (2.45GHz, 600 w)

(ii) Drying machines - textile, food & paper industry, for drying clothes, potato chips, printed matters etc.

(iii) Food processing industry - Precooling / coocking, pasteurising / sterility, heat frozen / refrigerated precooled meats, roasting of food grains / beans.

(iv) Rubber industry / plastics / chemical / forest product industries

(v) Mining / public works, breaking rock, tunnel boring, drying / breaking up concrete, breaking up coal seams, curing of cement.

(vi) Drying inks, drying / sterlising grains, drying / sterlising pharmaceuticals, drying textiles, leather, tobacco, power transmission.

(vii) Biomedical applications (diagnostic / therapeutic) - diathermy for localised superficial heating, deep electromagnetic heating for treatment of cancer, hyperthermia (local, regional or whole body for cancer therapy), electromagnetic transmission through human body has been used for monitoring of heart beat,

4. Radars:-

Detect aircraft, track/guide Supersonic missiles, observe & track weather patterns, Air Traffic Control (ATC), burglar alarms garage door openers, police speed detectors etc.

5. Remote Sensing:-

Many satellites are used to monitor the globe constantly for weather conditions, meteorology, ozone, soil moisture, agriculture, crop protection from frost, forests, snow thickr icebergs and other factors such as monitoring & exploration

6. Electronic warfare:-

ECM/ECCM (Electronic Counter Measure/Electronic Counter Counte Measure) systems, spread spectrum systems.

7. other Applications:-

The other applications include

→ Identifying objects & personnel by non-contact method.

→ Light generated charge carriers in a microwave semiconduc makes it possible to create a whole new world of microwave devices, fast jitter-free switches, phase shifters, HF generation, tuning elements etc.

Waveguides:-

At frequencies higher than 3GHz, transmission of electromagnetic waves along transmission lines & cables becom difficult mainly due to the losses that occur both in the solid dielectric needed to support the conductor & in the conductors themselves. A metallic tube can be used to transmit electromagnetic wave at these frequencies.

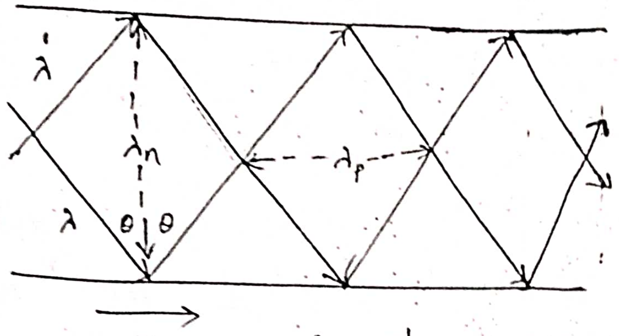
"A hallow metallic tube of uniform cross-section for transmitting electromagnetic waves by successive reflections from the inner walls of the tube is called a waveguide".

The most commonly used waveguides are

- Rectangular waveguide & • circular waveguides

Rectangular waveguides :-

"A rectangular waveguide is a hallow metallic tube with a rectangular cross-section."



λ_n - direction normal to Reflecting plane

λ_p - ||el to the plane

fig: Plane wave reflected in a waveguide.

The conducting walls of the guide confine the electromagnetic fields & there by guide the electromagnetic wave. when the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall. This process results in a component of either electric & magnetic field in the direction of propagation of the resultant wave. "Therefore the wave is no longer a Transverse ElectroMagnetic (TEM) wave."

Any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

1. Transverse electric mode :-

In this the electric field is transverse to the direction of propagation & has no component in that direction & there exist magnetic field component in the direction of propagation. such mode is called Transverse Electric (or) TE (or) H-mode.

2. Transverse Magnetic Mode :-

In this the magnetic field is transverse to the direction of propagation & has no component in that direction & there exist electric field component in the direction of propagation such mode is called Transverse Magnetic (or) TM (or) E-mode.

when the wavelength 'λ' is in the direction of propagation of the incident wave, there will be one component λ_n in the direction normal to the reflecting plane and another λ_p parallel to the plane. These components are

$$\lambda_n = \frac{\lambda}{\cos\theta} \qquad \lambda_p = \frac{\lambda}{\sin\theta}$$

where θ = angle of incidence

λ = wavelength of the impressed signal in unbounded medium

A planewave in a waveguide resolves into two components:

(1) standing wave in the direction normal to the reflecting walls of the guide

(2) A travelling wave in the direction parallel to the reflecting walls

In rectangular guides the modes are designated TE_{mn} & TM_{mn}

m = No. of halfwaves of electric & magnetic intensity in the x-direct

n = No. " " " y-directi

Here the propagation of wave is assumed in the +ve z direction

Solution of wave Equations in Rectangular Co-ordinates :-

The Electric & magnetic wave equations in frequency domain are given by

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

where γ = Propagation Constant.

$\gamma = \alpha + j\beta$

$= \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$

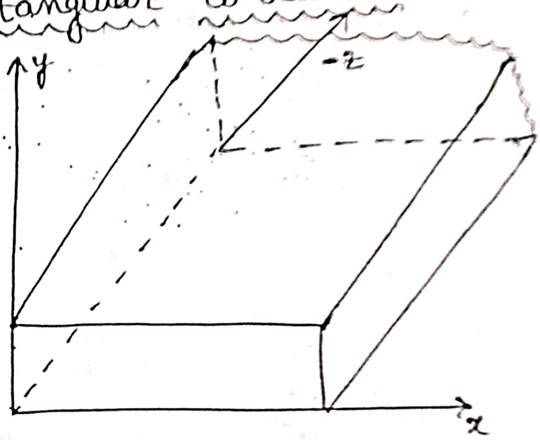


fig: Rectangular Co-ordinates

The rectangular components of E & H satisfy the complex scalar wave equation & Helmholtz equation

$$\nabla^2\psi = \gamma^2\psi$$

The Helmholtz equation in rectangular co-ordinates is given

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \gamma^2\psi \quad \text{---} (1)$$

This is a linear inhomogeneous partial differential equation in three dimensions. By the method of seperation of variables, the solution is assumed in the form of

$$\psi = X(x)\,Y(y)\,Z(z) \quad \text{---} (2)$$

where $X(x)$ = function of x coordinate only

$Y(y)$ = " y " "

$Z(z)$ = " z " "

Substituting ③ in ① & dividing the resultant by ③ we get

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \gamma^2$$

Since the sum of the three terms on the left hand side is a constant and each term is independently variable, it follows that each term must be equal to a constant.

let $\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \Rightarrow \frac{d^2X}{dx^2} = -k_x^2 X$ ——— ⓐ

$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 \Rightarrow \frac{d^2Y}{dy^2} = -k_y^2 Y$ ——— ⓑ

$\frac{1}{Z}\frac{d^2Z}{dz^2} = -k_z^2 \Rightarrow \frac{d^2Z}{dz^2} = -k_z^2 Z$ ——— ⓒ

$\therefore -k_x^2 - k_y^2 - k_z^2 = \gamma^2$

The general solution of each differential eqn ⓐ, ⓑ & ⓒ will be in the form

$$X = A\sin(k_x x) + B\cos(k_x x)$$

$$Y = C\sin(k_y y) + D\cos(k_y y)$$

$$Z = E\sin(k_z z) + F\cos(k_z z).$$

$\therefore$ The total solution is given by

$$\psi = [A\sin(k_x x) + B\cos(k_x x)][C\sin(k_y y) + D\cos(k_y y)][E\sin(k_z z) + F\cos(k_z z)]$$

The propagation constant γ_g in the waveguide differs from the intrinsic propagation constant γ of the dielectric.

$$\gamma_g^2 = \gamma^2 + k_x^2 + k_y^2 = \gamma^2 + k_c^2$$

where $\boxed{k_c = \sqrt{k_x^2 + k_y^2}}$ → cutoff wave number.

For a lossless dielectric $\sigma = 0 \Rightarrow \gamma = \sqrt{j\omega\mu(j\omega\epsilon)}$

$$= \sqrt{-\omega^2\mu\epsilon}$$

$$\gamma^2 = -\omega^2\mu\epsilon$$

$$\therefore \gamma_g = \pm\sqrt{-\omega^2\mu\epsilon + k_c^2}$$

(or) $$\gamma_g = \pm j\sqrt{\omega^2\mu\epsilon - k_c^2}$$

There are 3 cases for the Propagation constant γ_g in the waveguide.

case (i) :-

If $\omega^2\mu\epsilon = k_c^2 \Rightarrow \gamma_g = 0$

This is the critical condition for cutoff Propagation.

$\therefore$ The cutoff frequency is expressed as

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{k_x^2 + k_y^2}$$

$$f_c^2 = \frac{1}{(2\pi)^2\mu\epsilon}k_c^2$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}k_c$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{k_x^2 + k_y^2}$$

case (ii) :-

The wave will propagate when

$$\omega^2\mu\epsilon > k_c^2$$

$$\gamma_g = \pm j\beta_g = \pm j\omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\gamma_g = j\beta_g = j\sqrt{\omega^2\mu\epsilon\left(1 - \frac{k_c^2}{\omega^2\mu\epsilon}\right)}$$

$$= j\omega\sqrt{\mu\epsilon}\sqrt{1 - \frac{f_c^2(2\pi)^2}{(2\pi)^2 f^2}}$$

$$= j\omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

This means that the operating frequency must be above cutoff frequency inorder for a wave to Propagate in the guide.
ie, It acts as a highpass filter.

Case (iii) :-

The wave will be attenuated if

$$\omega^2\mu\epsilon < k_c^2$$

$$\gamma_g = \pm\alpha_g = \pm\omega\sqrt{\mu\epsilon}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

$$\gamma_g = \alpha_g = \pm\sqrt{\left(\frac{k_c^2}{\omega^2\mu\epsilon} - 1\right)\omega^2\mu\epsilon}$$

$$= \pm\omega\sqrt{\mu\epsilon}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

This means that if the operating frequency is below the cutoff frequency, the wave will decay exponentially with respect a factor of $-\alpha_g z$ & there will be no wave Propagation

...propagation constant is a real quantity.

∴ The solution to the Helmholtz equation in rectangular co-ordinates is given by

$$\psi = [A\sin(k_x x) + B\cos(k_x x)][C\sin(k_y y) + D\cos(k_y y)]\, e^{-j\beta_g z}.$$

TE mode in Rectangular waveguide :-

Here the wave is propagating in the positive z-direction in the waveguide.

The TE_{mn} mode in a rectangular wave guide are characterised by

$$\boxed{E_z = 0} \quad \text{———} (1)$$

H_z must exist inorder to have energy transmission in the guide.

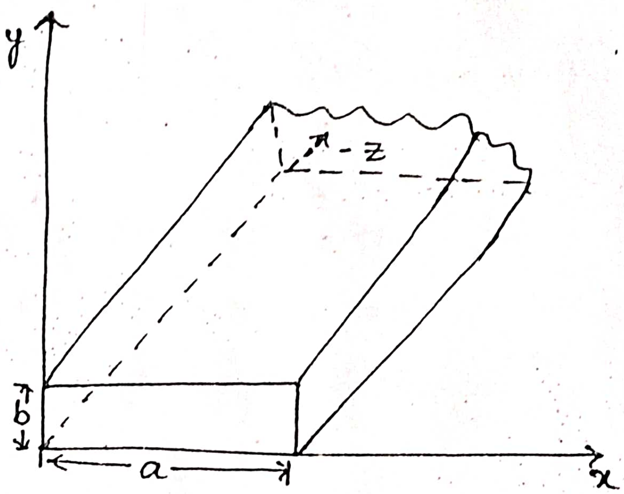


Fig: Co-ordinates of a rectangular waveguide

From Helmholtz equation

$$\nabla^2 H_z = \gamma^2 H_z.$$

The solution will be in the form of

$$H_z = [A\sin(k_x x) + B\cos(k_x x)][C\sin(k_y y) + D\cos(k_y y)]\, e^{-j\beta_g z}$$

will be determined in accordance with the given boundary conditions.

For a Lossless dielectric, Maxwell's curl equations in frequency domain are

$$\nabla \times E = \frac{-\partial B}{\partial t} = -j\omega\mu H$$

$$\nabla \times H = \frac{\partial D}{\partial t} = j\omega\epsilon E$$

In rectangular co-ordinates

$$\nabla \times E = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu\left[H_x i + H_y j + H_z k\right]$$

Comparing similar terms

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\nabla \times H = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

Substituting $\frac{\partial}{\partial z} = -j\beta_g$ & $E_z = 0$

$$\beta_g E_y = -\omega\mu H_x \quad \text{.......... (a)}$$

$$\beta_g E_x = \omega\mu H_y \quad \text{.......... (b)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \text{.......... (c)}$$

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega\epsilon E_x \quad \text{.......... (d)}$$

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad \text{.......... (e)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \quad \text{.......... (f)}$$

Solving the above six equations for E_x, E_y, H_x & H_y interm
... TE mode field equations in reclangul

from (b) $E_x = \frac{\omega\mu H_y}{\beta_g}$

from (d)

$$H_y = \left[j\omega\epsilon E_x - \frac{\partial H_z}{\partial y} \right] \frac{1}{j\beta_g}$$

$$\Rightarrow E_x = \frac{\omega\mu}{j\beta_g^2}\left[j\omega\epsilon E_x - \frac{\partial H_z}{\partial y} \right] = \frac{\omega^2\mu\epsilon E_x}{\beta_g^2} + \frac{j\omega\mu}{\beta_g^2}\frac{\partial H_z}{\partial y}$$

$$E_x\left[\frac{\omega^2\mu\epsilon}{\beta_g^2} - 1\right] = \frac{-j\omega\mu}{\beta_g^2}\frac{\partial H_z}{\partial y}$$

$$E_x\left[\frac{\omega^2\mu\epsilon - \beta_g^2}{\cancel{\beta_g^2}}\right] = \frac{-j\omega\mu}{\cancel{\beta_g^2}}\frac{\partial H_z}{\partial y}$$

$$k_c^2 = \gamma_g^2 - \gamma^2 = (j\beta_g)^2 - (-\omega^2\mu\epsilon) = -\beta_g^2 + \omega^2\mu\epsilon$$

$$E_x k_c^2 = -j\omega\mu\frac{\partial H_z}{\partial y}$$

$$\boxed{\therefore E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}}$$

From (a) $E_y = \frac{-\omega\mu H_x}{\beta_g}$

from (e)

$$H_x = \left[j\omega\epsilon E_y + \frac{\partial H_z}{\partial x} \right] \frac{1}{-j\beta_g}$$

$$E_y = \frac{-\omega\mu}{-j\beta_g^2}\left[j\omega\epsilon E_y + \frac{\partial H_z}{\partial x} \right] = \frac{\omega^2\mu\epsilon E_y}{\beta_g^2} - \frac{j\omega\mu}{\beta_g^2}\frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\omega^2\mu\epsilon}{\beta_g^2} - 1\right] = \frac{-j\omega\mu}{\beta_g^2}\frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\omega^2\mu\epsilon - \beta_g^2}{\beta_g^2}\right] = \frac{-j\omega\mu}{\beta_g^2}\frac{\partial H_z}{\partial x}$$

$$E_y k_c^2 = -j\omega\mu\frac{\partial H_z}{\partial x}$$

$$\boxed{\therefore E_y = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}}$$

from (d) & (e)

$$H_x = \left[j\omega\epsilon\left(\frac{-\omega\mu H_x}{\beta g}\right) + \frac{\partial H_z}{\partial x} \right] \frac{1}{-j\beta g}$$

$$= \frac{-j\omega^2\mu\epsilon H_x}{-j\beta g^2} + \frac{\partial H_z}{\partial x} \frac{1}{-j\beta g}$$

$$H_x\left[\frac{\omega^2\mu\epsilon}{\beta g^2} - 1\right] = \frac{\partial H_z}{\partial x} \frac{1}{j\beta g}$$

$$H_x\left[\frac{K_c^2}{\beta g^2}\right] = \frac{\partial H_z}{\partial x} \frac{1}{j\cancel{\beta g}}$$

$$\boxed{\therefore H_x = \frac{-j\beta g}{K_c^2} \frac{\partial H_z}{\partial x}}$$

from (b) & (d)

$$H_y = \left[j\omega\epsilon\left(\frac{\omega\mu H_y}{\beta g}\right) - \frac{\partial H_z}{\partial y} \right] \frac{1}{j\beta g}$$

$$= \frac{\omega^2\mu\epsilon H_y}{\beta g^2} - \frac{\partial H_z}{\partial y} \frac{1}{j\beta g}$$

$$H_y\left[\frac{\omega^2\mu\epsilon}{\beta g^2} - 1\right] = \frac{\partial H_z}{\partial y} \frac{1}{j\beta g}$$

$$H_y\left(\frac{\omega^2\mu\epsilon - \beta g^2}{\beta g^2}\right) = \frac{\partial H_z}{\partial y} \frac{1}{j\cancel{\beta g}}$$

$$H_y K_c^2 = -j\beta g \frac{\partial H_z}{\partial y}$$

$$\boxed{\therefore H_y = \frac{-j\beta g}{K_c^2} \frac{\partial H_z}{\partial y}}$$

The boundary conditions are

Either the tangent E-field or the normal H-field vanishes at the surface of the conductor.

$$\frac{\partial H_z}{\partial x} = k_x\left[A\cos(k_x x) - B\sin(k_x x)\right]\left[C\sin(k_y y) + D\cos(k_y y)\right]e^{-j\beta_g z}$$

$$\frac{\partial H_z}{\partial y} = k_y\left[A\sin(k_x x) + B\sin(k_x x)\right]\left[C\cos(k_y y) - D\sin(k_y y)\right]e^{-j\beta_g z}$$

(i) At $x=0$

$$E_y = 0 \Rightarrow \frac{\partial H_z}{\partial x} = 0$$

$$0 = A\cos(0) \Rightarrow \boxed{A=0}$$

(ii) At $x=a$

$$E_y = 0 \Rightarrow \frac{\partial H_z}{\partial x} = 0$$

$$0 = A\cos(k_x a) - B\sin(k_x a)$$

as $A=0$.

$$\sin(k_x a) = 0.$$

$$k_x a = \pm m\pi \quad , \quad m=1,2,3\ldots$$

$$\boxed{k_x = \pm\frac{m\pi}{a}}$$

(iii) At $y=0$

$$E_x = 0 \Rightarrow \frac{\partial H_z}{\partial y} = 0$$

$$0 = C\cos(0) \Rightarrow \boxed{C=0}$$

(iv) At $y=b$

$$E_x = 0 \Rightarrow \frac{\partial H_z}{\partial y} = 0$$

$$0 = C\cos(k_y b) - D\sin(k_y b)$$

as $C=0$.

$$\sin(k_y b) = 0.$$

$$k_y b = \pm n\pi \quad , \quad n=1,2,3\ldots$$

$$\boxed{\therefore k_y = \pm\frac{n\pi}{b}}$$

∴ The magnetic field in +ve z-direction is given by

$$H_z = H_{oz} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

where H_{oz} is the amplitude constant.

∴ The TE_{mn} field equations in the rectangular waveguide are given as

$$E_x = E_{ox} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$E_y = E_{oy} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$E_z = 0$$

$$H_x = H_{ox} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$H_y = H_{oy} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$H_z = H_{oz} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

where $m = 0, 1, 2 \ldots$

$n = 0, 1, 2 \ldots$

$m = n = 0$ excepted.

The cutoff wave number:-

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c \sqrt{\mu\epsilon}$$

where a & b are in meters

cutoff frequency:-

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{k_x^2 + k_y^2}$$

$$\Rightarrow f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Propagation constant (or) phase constant :-

$$\beta_g = \omega\sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

Phase velocity :-

"The rate at which the wave changes its phase interms of the guide wavelength is called phase velocity".

$$V_p = \frac{\omega}{\beta_g}$$

$$= \frac{1}{\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

Group velocity :-

"The rate at which the wave propogates through the wave guide is called group velocity".

$$V_g = \frac{d\omega}{d\beta_g}$$

$$\frac{d\beta_g}{d\omega} = \frac{d}{d\omega}\left[\omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}\right]$$

$$= \frac{d}{d\omega}\left[\sqrt{\mu\epsilon}\sqrt{\omega^2-\omega_c^2}\right]$$

$$= \sqrt{\mu\epsilon}\,\frac{\cancel{2}\omega}{\cancel{2}\sqrt{\omega^2-\omega_c^2}}$$

$$= \sqrt{\mu\epsilon}\,\frac{1}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

$$\therefore \frac{d\beta_g}{d\omega} = \frac{1}{c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{\therefore V_g = \frac{d\omega}{d\beta_g} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$V_g \cdot V_p = c\sqrt{1-\left(\frac{f_c}{f}\right)^2} \times \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{V_g \cdot V_p = c^2}$$

characteristic wave impedance :-

$$Z_g = \frac{E_x}{H_y} = \frac{-H_y}{E_x} = \frac{\omega\mu}{\beta_g} \qquad \frac{E_x}{H_y} = \frac{\omega\mu}{\beta_g}$$

$$= \frac{\omega\mu}{\omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \sqrt{\frac{\mu}{\epsilon}}\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\boxed{Z_g = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ = intrinsic impedance in unbounded dielectric.

wavelength in guide :-

"The distance required for phase to shift through 2π radians or $360°$".

$$\lambda_g = \frac{2\pi}{\beta_g}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$\lambda_g = \frac{c}{f}\frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} \Rightarrow \frac{2\pi}{\omega}$

$c = \frac{1}{\sqrt{\mu\epsilon}}$

$f = c/\lambda$

$\Rightarrow \lambda = c/f$

$$\boxed{\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}}$$

TM mode in Rectangular waveguide:

In TM mode if the wave is propagating along the positive z-direction

$$\boxed{H_z = 0} \quad \text{--- (1)}$$

From Helmholtz equation

$$\nabla^2 E_z = \gamma^2 E_z$$

The solution will be in the form of

$$E_z = \left[A\sin(k_x x) + B\cos(k_x x)\right]\left[C\sin(k_y y) + D\cos(k_y y)\right] e^{-j\beta_g z}$$

The maxwell's Equations in curl form are

$$\nabla \times E = -j\omega\mu H$$

$$\nabla \times H = j\omega\epsilon E$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial y} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

Substituting $\frac{\partial}{\partial z} = -j\beta_g$ & $H_z = 0$.

$$\frac{\partial E_z}{\partial y} + j\beta_g E_y = -j\omega\mu H_x \quad \text{......... (a)}$$

$$j\beta_g E_x + \frac{\partial E_z}{\partial x} = j\omega\mu H_y \quad \text{......... (b)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0 \quad \text{......... (c)}$$

$$\beta_g H_y = \omega \epsilon E_x \quad \ldots\ldots\ldots \text{(d)}$$

$$-\beta_g H_x = \omega \epsilon E_y \quad \ldots\ldots\ldots \text{(e)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \epsilon E_z \quad \ldots\ldots \text{(f)}$$

Solving the above six equations for E_x, E_y, H_x & H_y interms of E_z will give the TM mode field Equations in rectangular waveguide.

from (a) & (e)

from (e) :- $E_y = \dfrac{-\beta_g H_x}{\omega \epsilon}$

from (a)

$$\frac{\partial E_z}{\partial y} + j\beta_g \left[\frac{-\beta_g H_x}{\omega \epsilon} \right] = -j\omega \mu H_x$$

$$H_x \left[\frac{+j\beta_g^2}{\omega \epsilon} - j\omega \mu \right] = \frac{\partial E_z}{\partial y}$$

$$H_x \left[\frac{j\beta_g^2 - j\omega^2 \mu \epsilon}{\omega \epsilon} \right] = \frac{\partial E_z}{\partial y}$$

$$H_x = \frac{\omega \epsilon}{j(\beta_g^2 - \omega^2 \mu \epsilon)} \frac{\partial E_z}{\partial y} \qquad \because \beta_g^2 - \omega^2 \mu \epsilon = -K_c^2$$

$$= \frac{\omega \epsilon}{j(-K_c^2)} \frac{\partial E_z}{\partial y}$$

$$\boxed{H_x = \frac{j\omega \epsilon}{K_c^2} \frac{\partial E_z}{\partial y}}$$

from (e) $H_x = \dfrac{\omega \epsilon E_y}{-\beta_g}$

from (a)

$$\frac{\partial E_z}{\partial y} + j\beta_g E_y = -j\omega \mu \left(\frac{\omega \epsilon E_y}{-\beta_g} \right)$$

$$\frac{\partial E_z}{\partial y} = E_y \left(\frac{j\omega^2 \mu \epsilon}{\beta_g} - j\beta_g \right)$$

$$= E_y \left(\frac{j\omega^2 \mu \epsilon - j\beta_g^2}{\beta_g} \right)$$

$$\frac{\partial}{\partial y} = \ldots y \left(\frac{\quad}{\beta g} \right)$$

$$\boxed{E_y = \frac{-j\beta g}{k_c^2} \cdot \frac{\partial E_z}{\partial y}}$$

From (b) & (d)

from (d) $H_y = \frac{\omega \epsilon E_x}{\beta g}$

from (b) $j\beta g E_x + \frac{\partial E_z}{\partial x} = j\omega\mu \left[\frac{\omega \epsilon E_x}{\beta g} \right]$

$$\frac{\partial E_z}{\partial x} = E_x \left[\frac{j\omega^2 \mu \epsilon}{\beta g} - j\beta g \right]$$

$$= E_x \left[\frac{j(\omega^2 \mu \epsilon - \beta g^2)}{\beta g} \right]$$

$$= E_x \frac{k_c^2}{-j\beta g}$$

$$\boxed{\therefore E_x = \frac{-j\beta g}{k_c^2} \cdot \frac{\partial E_z}{\partial x}}$$

from (d) $E_x = \frac{H_y \beta g}{\omega \epsilon}$

from (b) $j\beta g \left[\frac{H_y \beta g}{\omega \epsilon} \right] + \frac{\partial E_z}{\partial x} = j\omega\mu H_y$

$$\frac{\partial E_z}{\partial x} = H_y \left[j\omega\mu - \frac{j\beta g^2}{\omega \epsilon} \right]$$

$$= H_y \left[\frac{j(\omega^2 \mu \epsilon - \beta g^2)}{\omega \epsilon} \right]$$

$$= H_y \left[\frac{k_c^2}{-j\omega \epsilon} \right]$$

$$\boxed{H_y = \frac{-j\omega \epsilon}{k_c^2} \frac{\partial E_z}{\partial x}}$$

The boundary conditions are Electric field vanishes at Surface of conductor.

$$\frac{\partial E_z}{\partial x} = k_x\left[A\cos(k_x x) - B\sin(k_x x)\right]\left[C\sin(k_y y) + D\cos(k_y y)\right] e^{-j\beta_g z}$$

$$\frac{\partial E_z}{\partial y} = k_y\left[A\sin(k_x x) + B\cos(k_x x)\right]\left[C\cos(k_y y) - D\sin(k_y y)\right] e^{-j}$$

(i) At $x=0$

$$E_y = 0 \Rightarrow \frac{\partial E_z}{\partial y} = 0$$

$$0 = B\cos 0 \Rightarrow \boxed{B=0}$$

(ii) At $x=a$

$$E_y = 0 \Rightarrow \frac{\partial E_z}{\partial y} = 0$$

$$0 = A\sin(k_x a) + B\cos(k_x a)$$

as $B=0$

$$\sin(k_x a) = 0$$

$$k_x a = \pm m\pi \quad , \quad m = 1, 2, \ldots$$

$$\boxed{\therefore k_x = \frac{\pm m\pi}{a}}$$

(iii) At $y=0$

$$E_x = 0 \Rightarrow \frac{\partial E_z}{\partial x} = 0$$

$$0 = D\cos 0 \Rightarrow \boxed{D=0}$$

(iv) At $y=b$

$$E_y = 0 \Rightarrow \frac{\partial E_z}{\partial x} = 0$$

$$0 = C\sin(k_y b) + D\cos(k_y b)$$

as $D=0$

$$\sin(k_y b) = 0 \Rightarrow k_y b = \pm n\pi \quad , \quad n = 0, 1, 2 \ldots$$

$$\boxed{\therefore k_y = \frac{\pm n\pi}{b}}$$

∴ The electric field in the +ve z-direction is given by

$$E_z = E_{oz} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}\right) y\, e^{-j\beta_g z}$$

where E_{oz} is the amplitude constant.

∴ The TM_{mn} field equations in rectangular waveguide are given as

$$E_x = E_{ox} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$E_y = E_{oy} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$E_z = E_{oz} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$H_x = H_{ox} \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$H_y = H_{oy} \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta_g z}$$

$$H_z = 0.$$

where $m = 1, 2, 3 \ldots$

$n = 1, 2, 3 \ldots$

If either $m=0$ or $n=0$ the field intensities all vanish. So there is no TM_{01} or TM_{10} mode in rectangular waveguide.

Cutoff wavenumber:-

$$k_c = \sqrt{k_x^2 + k_y^2} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c\sqrt{\mu\epsilon}$$

cutoff frequency:-

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Propagation or phase constant:-

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

phase velocity

$$v_p = \frac{\omega}{\beta_g} = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

Group velocity.

$$v_g = \frac{d\omega}{d\beta_g} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

characteristic wave impedance:-

$$Z_{TM} = \frac{\beta_g}{\omega \epsilon} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

wavelength in guide:-

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Degenerate Modes:-

"If two or more modes have the same cutoff frequency, they are said to be degenerate modes".

In a rectangular waveguide TE_{mn}, TM_{mn} modes are always degenerate.

Ex: TE_{11}, TM_{11} $\Rightarrow f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}$

TE_{12}, TM_{12}

Dominant Modes:-

The mode with the lowest cutoff frequency in a particular guide is called the dominant mode.

The dominant mode in a rectangular waveguide with $a>b$ is "TE_{10} mode".

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{0}{b}\right)^2}$$

$$f_c = \frac{c}{2a}$$

In TM_{mn} mode TM_{11} is the dominant mode.

do not exist.

(Q) Derive the expression for cutoff frequency, phase constant, group velocity, phase velocity and wave impedance in a rectangular waveguide.

(Q) Starting with the equation for the propagation constant of a mode in rectangular waveguide. Derive the expression

$$\lambda_0 = \frac{\lambda_g \lambda_c}{\sqrt{\lambda_g^2 + \lambda_c^2}}$$

where λ_g = guide wavelength

λ_c = cutoff wavelength

λ_0 = free space wave length

The guide wavelength

$$\lambda_g = \frac{2\pi}{\beta_g}$$

$$= \frac{2\pi}{\omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{\cancel{2\pi} \times c}{\cancel{2\pi} f\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}}$$

$$\left(\frac{\lambda_g}{\lambda_0}\right)^2 = \frac{\lambda_c^2}{\lambda_c^2 - \lambda_0^2}$$

$$\lambda_g^2\lambda_c^2 - \lambda_g^2\lambda_0^2 = \lambda_0^2\lambda_c^2$$

$$\lambda_g^2\lambda_c^2 = \lambda_0^2\left[\lambda_c^2 + \lambda_g^2\right]$$

$$\lambda_0^2 = \frac{\lambda_g^2\lambda_c^2}{\lambda_c^2 + \lambda_g^2}$$

$$\boxed{\therefore \lambda_0 = \frac{\lambda_g \lambda_c}{\sqrt{\lambda_c^2 + \lambda_g^2}}}$$

Power Transmission in Rectangular waveguides (13)

The Power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of the comple poynting theorem. It is assumed that the guide is terminated insuchaway that there is no reflection from the receiving end or that the guide is infinitely long compared with the wavelength or that the guide is

From the poynting theorem the power transmitted through a guide is given by

$$P_{tr} = \oint P \cdot ds = \oint \frac{1}{2}(E \times H^*) \cdot ds$$

For a lossless dielectric, the time-average power flow through a rectangular guide is given by

$$P_{tr} = \frac{1}{2 Z_g} \int_a |E|^2 da = \frac{Z_g}{2} \int_a |H|^2 da \quad \text{— (a)}$$

where $Z_g = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

For TE_{mn} modes, the average power transmitted through a rectangular waveguide is given by

$$P_{tr} = \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{2\eta} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\, dy$$

For TM_{mn} modes, the average power transmitted through a rectangular waveguide is given by

$$P_{tr} = \frac{1}{2\eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dx\, dy.$$

Power losses in a rectangular waveguide:

There are two types of power losses in a rectangular waveguide

(1) Losses in the dielectric

(2) Losses in the guide walls.

(1) Let us consider the power losses caused by dielectric attenuation. In a lowloss dielectric ($\sigma << \omega\varepsilon$), the propagation constant for a plane wave travelling in an unbounded lossy dielectric is given by

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta\sigma}{2} \qquad \text{——— (1)}$$

The attenuation caused by the low-loss dielectric in the rectangular waveguide is given by

$$\alpha_g = \frac{\sigma\eta}{2\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \text{for TE mode ——— (2)}$$

$$\alpha_g = \frac{\sigma\eta}{2}\sqrt{1-\left(\frac{f_c}{f}\right)^2} \qquad \text{for TM mode ——— (3)}$$

If $f >> f_c$, the attenuation constant in the guide approaches that for the unbounded dielectric as in Eqn (1).

If $f << f_c$, The attenuation constant becomes very large & non-propagation occurs.

(2) Let us consider the losses caused by the guide walls. when the electric & magnetic intensities propagate through a lossy waveguide, their magnitudes may be written

$$|E| = |E_{oz}|\, e^{-\alpha_g z}$$

$P_{AEF} = |E_{oz}|^2 e^{-2\alpha_g z}$

$$|H| = |H_{oz}|\, e^{-\alpha_g z}$$

where E_{oz} & H_{oz} are the field intensities at $z=0$.

For low loss guide, the time average power flow decreases proportionally to $e^{-2\alpha_g z}$. Hence

$$P_{tr} = (P_{tr} + P_{loss})\, e^{-2\alpha_g z}$$

$$(1 - e^{-2\alpha_g z})\, P_{tr} = P_{loss}\, e^{-2\alpha_g z}$$

$$\frac{P_{loss}}{P_{tr}} = \frac{1}{e^{-2\alpha_g z}} - 1$$

$$\frac{P_{loss}}{P_{tr}} + 1 = e^{2\alpha_g z}$$

since $P_{loss} << P_{tr}$ and $2\alpha_g z << 1$.

$$\frac{P_{loss}}{P_{tr}} + 1 = 1 + 2\alpha_g z$$

$$\alpha_g = \frac{P_{loss}}{2P_{tr} z}$$

$$\boxed{\alpha_g = \frac{P_L}{2P_{tr} z}} \quad -(2)$$

where P_L = Power loss per unit length

The electric and magnetic field intensities established at The surface of low-loss guide wall decay exponentially w.r.t The Skin depth while the wave propagates into the walls, it is better to define a surface resistance of the guide walls as

$$R_S = \frac{\rho}{\delta} = \frac{1}{\sigma\delta} = \frac{\sqrt{\pi\mu f \sigma}}{\sigma} = \sqrt{\frac{\pi\mu f}{\sigma}} \ \Omega/\text{square}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$ where ρ = Resistivity of The conducting wall in Ω-m

σ = conductivity in $\mho$/m

δ = Skin depth & depth of penetration in m.

The Power loss per unit length of guide is obtained by integrating the power density over the surface of the conductor corresponding to The unit length of the guide.

$$P_L = \frac{R_s}{2} \int |H_t|^2 \, ds \quad \text{W/unit length.} \quad -(3)$$

where H_t is the tangential component of the magnetic intensity at the guide walls.

$$\therefore \alpha_g = \frac{R_s \int_s |H_t|^2 ds}{2 Z_g \int_a |H|^2 da}$$

substitute eq^n ①, ② in eq^n ④

where

$$|H|^2 = |H_x|^2 + |H_y|^2$$

$$|H_t|^2 = |H_{tx}|^2 + |H_{ty}|^2.$$

Problems :-

① A rectangular waveguide is filled by dielectric material of $\epsilon_r = 9$ and has dimensions of 7×3.5cm. It operates in the dominant TE mode.

(i) Determine the cutoff frequency

(ii) Find the phase velocity in the guide at a frequency of 2GHz

(iii) Find the guide wavelength.

Given data $a = 7$cm., $b = 3.5$cm

$\epsilon_r = 9$.

The dominant mode in TE is TE_{10} i.e, $m=1$, $n=0$.

(i) $$f_c = \frac{1}{2\sqrt{\mu_0 \epsilon_0 \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{4\pi \times 10^{-7} \times 9 \times \frac{1}{36\pi \times 10^9}}} \times \left[\sqrt{\left(\frac{1}{7 \times 10^{-2}}\right)^2}\right]$$

$\mu = 4\pi \times 10^{-7}$

$\epsilon_0 = \frac{1}{36\pi \times 10^9}$

$$\boxed{f_c = 0.714 GHz}$$

(ii) $$V_p = \frac{1}{\sqrt{\mu \epsilon_0 \epsilon_r} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

$f = 2GHz$, $f_c = 0.714GHz$.

$$= \frac{1}{\sqrt{4\pi \times 10^{-7} \times 9 \times \frac{1}{36\pi \times 10^9}} \sqrt{1 - \left(\frac{0.714G}{2G}\right)^2}}$$

$$\boxed{V_p = 1.07 \times 10^8 \ m/sec}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad \lambda = c/f$$

$$= \frac{\frac{3\times10^8}{2\times10^9}}{\sqrt{1-\left(\frac{0.714}{2}\right)^2}}$$

$$\boxed{\lambda_g = 0.16\ m}$$

(2) A rectangular waveguide with dimension of 3x2cm operates in the TM_{11} mode at 10GHz. Determine the characteristic wave impedance.

Given data

$a = 3cm = 3\times10^{-2}m$

$b = 2cm = 2\times10^{-2}m$

$TM_{11} \Rightarrow m=1, n=1$

$f = 10GHz$

$Z_{TM} = ?$

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{1}{3\times10^{-2}}\right)^2+\left(\frac{1}{2\times1}\right.}$$

$$\boxed{f_c = 9.013GHz.}$$

$$Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= 120\pi\sqrt{1-\left(\frac{8.08}{10}\right)^2}$$

$$\boxed{Z_{TM} = 163.62\Omega}$$

(3) An airfilled rectangular waveguide has dimensions of a=6cm, b=4cm. The signal frequency is 3GHz. Compute the following for TE_{10}, TE_{11} modes.

(i) cut-off frequency

(ii) wavelength in waveguide

(iii) phase constant & phase velocity in the waveguide

(iv) Group velocity & wave impedance in the waveguide.

given data

$a = 6cm = 6\times10^{-2}m$

$b = 4cm = 4\times10^{-2}m$

$f = 3GHz$

TE$_{10}$ mode

$m = 1, \quad n = 0.$

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{100}{6}\right)^2+\left(\frac{100\times0}{4}\right)^2}$$

(i) $\boxed{f_c = 2.5GHz}$

wavelength in the waveguide

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}} = \frac{3\times10^8/3\times10^9}{\sqrt{1-\left(\frac{2.5}{3}\right)^2}} = 0.18\,m$$

$\boxed{\lambda_g = 18cm}$

phase constant & phase velocity :-

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$\omega = 2\pi f = 2\pi\times 3G$

$$= \frac{2\pi\times3\times10^9}{3\times10^8}\sqrt{1-\left(\frac{2.5G}{3G}\right)^2}$$

$\beta = 34.67$ radians/m

$$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^8}{\sqrt{1-\left(\frac{2.5}{3}\right)^2}} = 5.4\times10^8\,m/sec$$

(iv) Group velocity :-

$$v_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= 3\times10^8\sqrt{1-\left(\frac{2.5}{3}\right)^2}$$

$$\boxed{v_g = 1.65\times10^8 \ m/sec}$$

wave impedance

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{120\pi}{\sqrt{1-\left(\frac{2.5}{3}\right)^2}}$$

$$\boxed{Z_{TE} = 681.97 \ \Omega}$$

TE_{11} mode :-

$m=1, n=1$.

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times10^8}{2}\sqrt{\left(\frac{100}{6}\right)^2+\left(\frac{100}{4}\right)^2}$$

$$\boxed{f_c = 4.5 \ GHz}$$

Given $f = 3GHz$.

Here $f < f_c$

$\therefore$ TE_{11} mode is not possible.

(4) A rectangular waveguide has $a = 4cm$, $b = 3cm$ as its sectional dimensions. Find all the modes which will propagate at 500 MHz.

$f = 500 \ MHz = 0.5 GHz$

$a = 4\times10^{-2}$ m.

$b = 3\times10^{-2}$ m.

for TE_{10} mode.

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{2}\sqrt{\left(\frac{100}{4}\right)^2} = 3.75\,GHz.$$

Here $f < f_c \Rightarrow TE_{10}$ mode does not exist.

As TE_{10} mode is the dominant mode in the rectangular waveguide as it is not possible with the given specifications, no other modes are possible.

(5) A rectangular waveguide has a cross section of 1.5×0.8 cm, $\sigma=0$, $\mu=\mu_0$ & $\epsilon=4\epsilon_0$. The magnetic field component is given by

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\pi\times10^{11}t-\beta z\right)\ A/m.$$

Determine

(i) The mode of Propagation

(ii) Cut off frequency.

(iii) phase constant

(iv) Propagation Constant.

(v) wave impedance.

Sol Given data.

$$H_x = 2\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{3\pi y}{b}\right)\sin\left(\pi\times10^{11}t-\beta z\right)\ A/m.$$

Comparing with

$$H_x = H_{0x}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t-\beta z)\ A/m.$$

$m=1,\ n=3.\quad \omega = \pi\times10^{11} \Rightarrow f = \frac{\omega}{2\pi} = \frac{\pi\times10^{11}}{2\pi} = 50\,GHz$

$\therefore$ The mode of operation is TE_{13} or TM_{13}

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{4\mu_0\epsilon_0}}\sqrt{\left(\frac{100}{1.5}\right)^2+\left(\frac{100}{0.8}\right)^2}$$

$$\boxed{f_c = 28.57\ GHz}$$

(iii) phase constant

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \pi\times10^{11}\sqrt{4\mu_0\epsilon_0}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$= \frac{2\pi\times10^{11}}{3\times10^8}\sqrt{1-\left(\frac{28.57}{50}\right)^2}$$

$$\boxed{\beta_g = 1718.81\ rad/m}$$

(iv) Propagation constant

$$\gamma = j\beta_g = 1718.81\ rad/m$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

(v) for TE_{13} mode

$$Z_{TE} = \frac{\eta/2}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{120\pi}{\sqrt{1-\left(\frac{28.57}{50}\right)^2}} = 229.69\ \Omega$$

for TM_{13} mode

$$Z_{TM} = \eta/2\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 120\pi\sqrt{1-\left(\frac{28.57}{50}\right)^2} = 309.38\ \Omega$$

(6) A 6GHz signal is to be propaged in the dominant mode in a rectangular waveguide. If its group velocity is to be 80% of the free space velocity of light. what must be the

to this signal if it is correctly matched.

Given data:-

$f = 6GHz$

Dominant mode in rectangular waveguide $= TE_{10} \Rightarrow m=1, n=0$.

$$V_g = \frac{80}{100} \times C = 0.8C$$

Breadth (a) : ?

$Z_{TE} = ?$

$$V_g = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

$$0.8\cancel{c} = \cancel{c}\sqrt{1-\left(\frac{f_c}{6G}\right)^2}$$

$$0.64 = 1 - \frac{f_c^2}{36 \times 10^{18}}$$

$$\boxed{f_c = 3.6GHz}$$

$$f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$3.6G = \frac{3 \times 10^8}{2}\left(\frac{1}{a}\right)$$

$$a = 0.0416\,m$$

$$a = 4.167\,cm$$

$$\boxed{\therefore a \approx 4\,cm}$$

$\therefore$ The impedance offered to the applied signal

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 471.23\,\Omega$$

$$\boxed{\therefore Z_{TE} = 471.23\,\Omega}$$

Filter characteristics:-

The waves which have frequencies greater than the cutoff frequencies of the waveguide can only be propagated through a waveguide. Hence a waveguide can be treated as a High Pass filter.