UNIT – 3 UNIT-III: ELECTRICAL MEASUREMENTS: *Measurement of Voltage and Current*: D‘Arsonval Galvanometer, permanent magnet moving coil, permanent magnet moving iron, Dynamometer, *Measurement of Resistance, Inductance and Capacitance*: Wheat stone bridge, Kelvin double bridge, Wien Bridge, Hay‘s bridge, Maxwell bridge, Anderson bridge, Q- Meter, Schering bridge, Ohmmeter.

INTRODUCTION: Analogue ammeters and voltmeters are classified together, since there is no basic difference in their operating principles. The action of all ammeters and voltmeters, except those of the electrostatic variety, depends upon a deflecting torque produced by an electric current. In an ammeter this torque is produced by the current to be measured, or by a definite fraction of it. In a voltmeter it is produced by a current that is proportional to the voltage to be measured. Hence both voltmeters and ammeters are essentially current measuring devices. The essential requirements of a measuring instrument are (a) that its introduction into the circuit where measurements are to be made, should not alter the circuit conditions, and (b) the power consumed by it be small.

D‘ARSONVAL GALVANOMETER or BASIC METER MOVEMENT

The action of the most commonly dc meter is based on the fundamental principle of the motor. The motor action is produced by the fl ow of a small current through a moving coil, which is positioned in the field of a permanent magnet. This basic moving coil system is often called the D’Arsonval galvanometer.

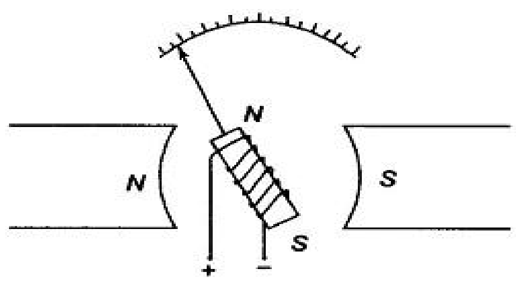


Fig. 2.1 — D’Arsonval Principle

The D’Arsonval movement shown in Fig. 2.1 employs a spring-loaded coil through which the measured cur rent fl ows. The coil (rotor) is in a nearly homogeneous field of a permanent mag net and moves in a rotary fashion. The amount of rotation is proportional to the amount of current fl owing through the coil. A pointer attached to the coil indicates the position of the coil on a scale calibrated in terms of current or voltage. It responds to dc current only, and has an almost linear calibration. The magnetic shunt that varies the field strength is used for calibration.

PERMANENT MAGNET MOVING COIL GALVANOMETER:

The basic PMMC movement (also called a D’Arsonval movement) offers the largest magnet in a given space, in the form of a horse-shoe, and is used when a large flux is required in the air gap. The D'Arsonval movement is based on the principle of a moving electromagnetic coil pivoted in a uniform air gap between the poles of a large fixed permanent magnet. This principle is illustrated in Fig. 2.1 With the polarities as shown, there is a repelling force between like poles, which exerts a torque on the pivoted coil. The torque is proportional to the magnitude of current being measured. This D'Arsonval movement provides an instrument with very low power consumption and low current required for full scale deflection (fsd).

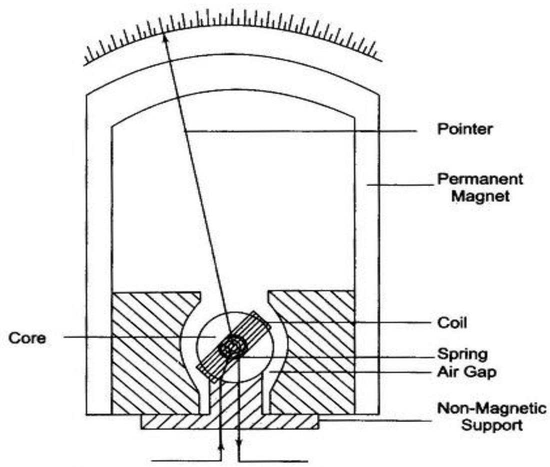


Fig. 2.2 Modern D'Arsonval Movement

Figure 2.2 shows a permanent horse-shoe magnet with soft iron pole pieces attached to it. Between the pole pieces is a cylinder of soft iron which serves to provide a uniform magnetic field in the air gap between the pole pieces and the cylindrical core. The coil is wound on a light metal frame and is mounted so that it can rotate freely in the air gap. The pointer attached to the coil moves over a graduated scale and indicates the angular defl ection of the coil, which is proportional to the current flowing through it.

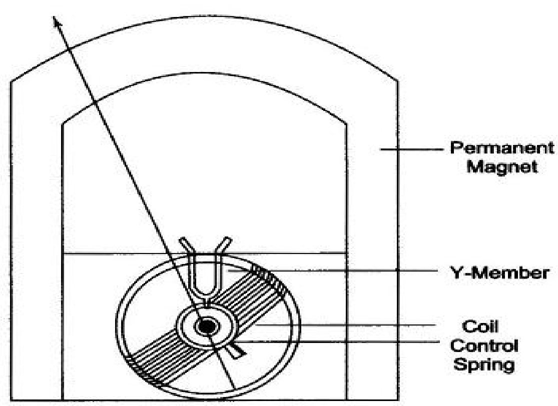


Fig. 2.3 Simplified Diagram of a PMMC Movement Showing the Y-member

The Y-shaped member shown in Fig. 2.3 is the zero adjust control, and is connected to the fixed end of the front control spring. An eccentric pin through the instrument case engages the Y-shaped member so that the zero position of the pointer can be adjusted from outside. The calibrated force opposing the moving torque is provided by two phosphor-bronze conductive springs, normally equal in strength. (This also provides the necessary torque to bring the pointer back to its original position after the measurement is over.)

The accuracy of the instrument can be maintained by keeping spring performance constant. The entire moving system is statically balanced at all positions by three (counterweights) balance weights. The pointer, springs, and pivots are fixed to the coil assembly by means of pivot bases and the entire movable coil element is supported by jewel bearings. PMMC instruments are constructed to produce as little viscious damping as possible and the required degree of damping is added.

Advantages

1. They can be modified with the help of shunts and resistance to cover a wide range of currents and voltages.
2. They display no hystersis
3. Since operating fields of such instruments are very strong, they are not significantly affected by stray magnetic fields.

Disadvantages

1. Some errors may set in due to ageing of control springs and the permanent magnet.
2. Friction due to jewel-pivot suspension.

<u>PERMANENT MAGNET MOVING IRON GALVANOMETER</u>

Moving iron instruments can be classified into attraction and repulsion types. Repulsion type instruments are the most commonly used. Iron vane ammeters and voltmeters depend for their operations on the repulsion that exists between two like magnetic poles.

The movement consists of a stationary coil of many turns which carries the current to be measured. Two iron vanes are placed inside the coil. One vane is rigidly attached to the coil frame, while the other is connected to the instrument shaft which rotates freely. The current through the coil magnetises both the vanes with the same polarity, regardless of the instantaneous direction of current. The two magnetised vanes experience a repelling force, and since only one vane can move, its displacement is an indicator of the magnitude of the coil current. The repelling force is proportional to the current squared, but the effects of frequency and hysteresis tend to produce a pointer deflection that is not linear and that does not have a perfect square law relationship.

Figure 2.8 shows a radial vane repulsion instrument which is the most sensitive of the moving iron mechanisms and has the most linear scale. One of these like poles is created by the instrument coil and appears as an iron vane fixed in its position within the coil, as shown in Fig. 2.8. The other like pole is induced on the movable iron piece or vane, which is suspended in the induction field of the coil and to which the needle of the instrument is attached. Since the instrument is used on ac, the magnetic polarity of the coil changes with every half cycle and induces a corresponding amount of repulsion of the movable vane against the spring tension. The defl ection of the instrument pointer is therefore always in the same direction, since there is always repulsion between the like poles of the fixed and the movable vane, even though the current in the inducing coil alternates.

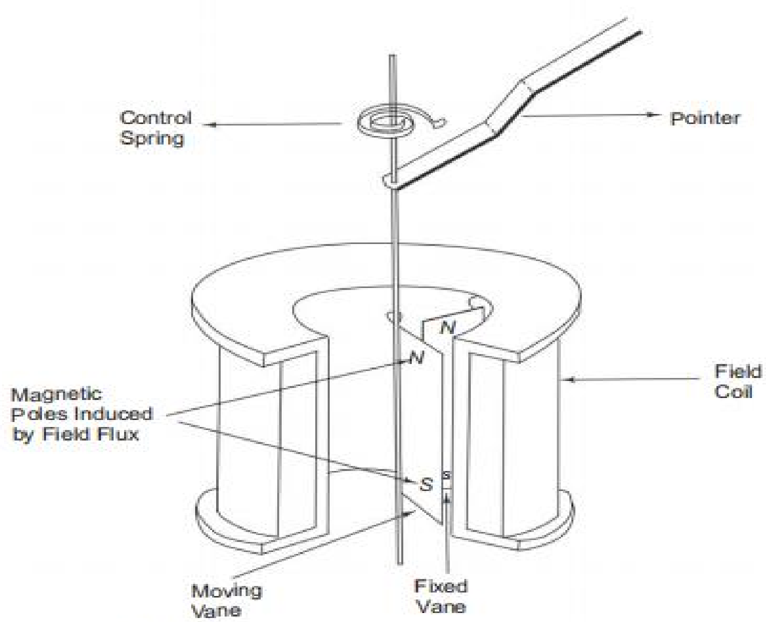


Fig. 2.8 Repulsion type AC meter (Radial vane type)

The deflection of the pointer thus produced is effectively proportional to the actual current through the instrument. It can therefore be calibrated directly in amperes and volts. The calibrations of a given instrument will however only be accurate for the ac frequency for which it is designed, because the impedance will be different at a new frequency The moving coil or repulsion type of instrument is usually calibrated to read the effective value of amperes and volts, and is used primarily for rugged and inexpensive meters.

The iron vane or radial type is forced to turn within the fixed current carrying coil by the repulsion between like poles. The aluminium vanes, attached to the lower end of the pointer, acts as a damping vane, in its close fitting chamber, to bring the pointer quickly to rest.

ELECTRODYNAMOMETER or DYNAMOMETER

The D'Arsonval movement responds to the average or dc value of the current fl owing through the coil. If ac current is sought to be measured, the current would fl ow through the coil with positive and negative half cycles, and hence the driving torque would be positive in one direction and negative in the other. If the frequency of the ac is very low, the pointer would swing back and forth around the zero point on the meter scale. At higher frequencies, the inertia of the coil is so great that the pointer does not follow the rapid variations of the driving torque and vibrates around the zero mark.

Therefore, to measure ac on a D'Arsonval movement, a rectifier has to be used to produce a unidirectional torque. This rectifier converts ac into dc and the rectified current deflects the coil. Another method is to use the heating effect of ac current to produce an indication of its magnitude. This is done using an electrodynamometer (EDM). An electrodynamometer is often used in accurate voltmeter and ammeters not only at power line frequency but also at low AF range. The electrodynamometer can be used by slightly modifying the PMMC movement. It may also serve as a transfer instrument, because it can be calibrated on dc and then used directly on ac thereby equating ac and dc measurements of voltage and current directly. A movable coil is used to provide the magnetic field in an electrodynamometer, instead of a permanent magnet, as in the D'Arsonval movement. This movable coil rotates within the magnetic field. The EDM uses the current under measurement to produce the required field flux.

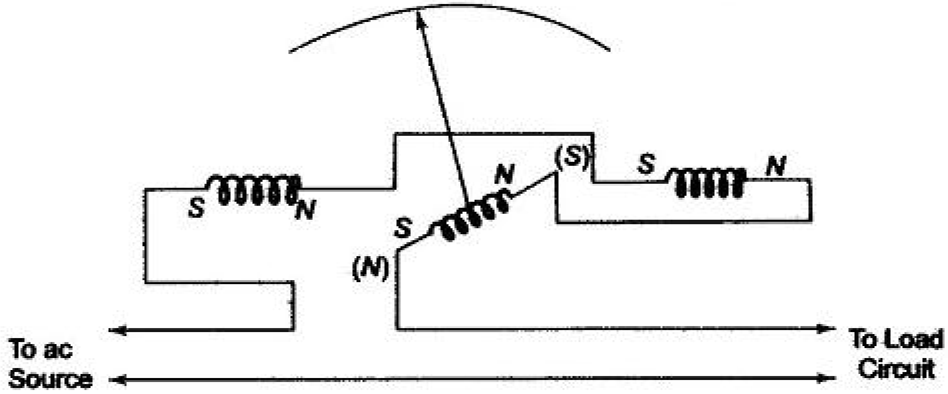


Fig. 2.6 (a) Basic EDM as an Ammeter

A fixed coil, split into two equal halves provides the magnetic field in which the movable coil rotates, as shown in Fig. 2.6 (a). The coil halves are connected in series with the moving coil and are fed by the current being measured. The fixed coils are spaced far apart to allow passage for the shaft of the movable coil. The movable coil carries a pointer, which is balanced by counterweights. Its rotation is controlled by springs, similar to those in a D'Arsonval movement. The complete assembly is surrounded by a laminated shield to protect the instrument from stray magnetic field which may affect its operation. Damping is provided by aluminium air vanes moving in a sector shaped chamber. (The entire movement is very solid and rigidly constructed in order to keep its mechanical dimensions stable, and calibration intact.)

The operation of the instrument may be understood from the expression for the torque developed by a coil suspended in a magnetic field, i.e.

$$\tau = B \times A \times N \times I$$

indicating that the torque which deflects the movable coil is directly proportional to the coil constants (A and N), the strength of the magnetic field in which the coil moves (B), and the current (I) flowing through the coil. In an EDM the flux density (B) depends on the current through the fixed coil and is therefore proportional to the deflection current (I). Since the coil constants are fixed quantities for any given meter, the developed torque becomes a function of the current squared (I^2).

If the EDM is used for dc measurement, the square law can be noticed by the crowding of the scale markings at low current values, progressively spreading at higher current values. For ac measurement, the developed torque at any instant is proportional to the instantaneous current squared (i^2). The instantaneous values of i^2 are always positive and torque pulsations are therefore produced. The meter movement, however, cannot follow rapid variations of the torque and take up a position in which the average torque is balanced by the torque of the control springs. The meter deflection is therefore a function of the mean of the squared current.

The scale of the EDM is usually calibrated in terms of the square root of the average current squared, and therefore reads the effective or rms value of the ac. The transfer properties of the EDM become apparent when we compare the effective value of the alternating current and the direct current in terms of their heating effect, or transfer of power.

The EDM has the disadvantage of high power consumption, due to its construction. The current under measurement must not only pass through the movable coil, but also provide the necessary field flux to get a sufficiently strong magnetic field. Hence high mmf is required and the source must have a high current and power. In spite of this high power consumption the magnetic field is still weaker than that of the D'Arsonval movement because there is no iron in the path, the entire flux path consisting of air. The EDM can be used to measure ac or dc voltage or current, as shown in Figs. 2.6 (a) and (b). Typical values of EDM flux density are in the range of approximately 60 gauss as compared to the high flux densities (1000 – 4000 guass) of a good D'Arsonval movement.

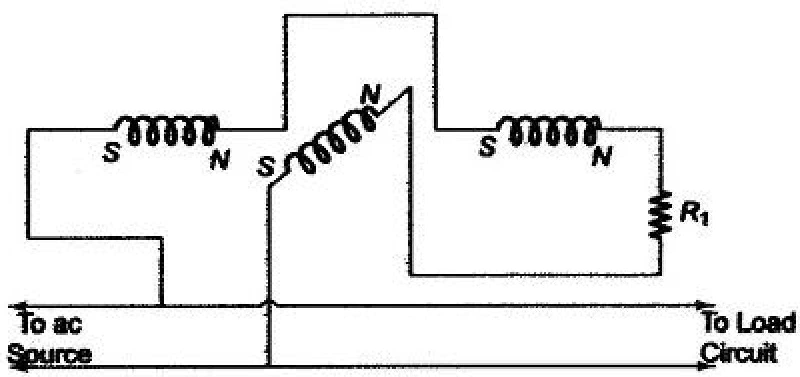


Fig. 2.6 (b) Basic EDM as a Voltmeter

The low flux density of the EDM affects the developed torque and therefore the sensitivity of the instrument. The addition of a series multiplier converts the basic EDM into a voltmeter [Fig. 2.6 (b)] which can be used for ac and dc measurements. The sensitivity of the EDM voltmeter is low, approximately 10 – 30 W/V, compared to 20 kW/V of the D'Arsonval movement. It is however very accurate at power line frequency and can be considered as a secondary standard. The basic EDM shown in Fig. 2.6 (a) can be converted into an ammeter (even without a shunt), because it is difficult to design a moving coil which can carry more than approximately 100 mA. The EDM movement is extensively used to measure power, both dc and ac, for any waveform of voltage and current. An EDM used as a voltmeter or ammeter has the fixed coils and movable coil connected in series, thereby reacting to I^2 .

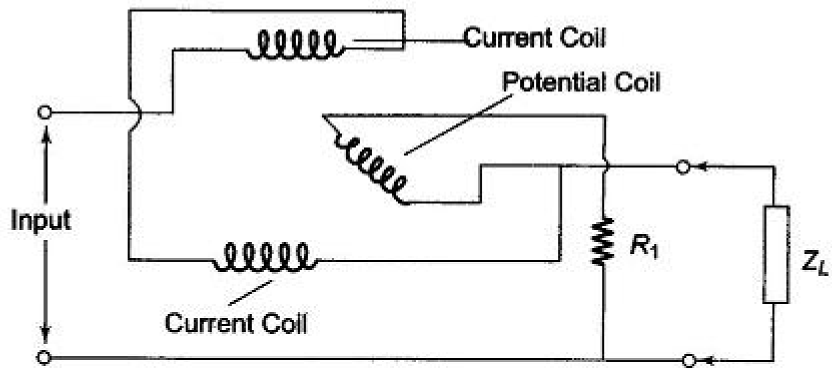


Fig. 2.7 EDM as a Wattmeter

When an EDM is used as a single phase wattmeter, the coil arrangement is different, as shown in Fig. 2.7. The fixed coils, shown in Fig. 2.7 as separate elements, are connected in series and carries the total line current. The movable coil located in the magnetic field of the fixed coils is connected in series with a current-limiting resistor across the power line, and carries a small current. The deflection of the movable coil is proportional to the product of the instantaneous value of current in the movable coil and the total line current. The EDM wattmeter consumes some power for the maintenance of its magnetic field, but this is usually small compared to the load power.

PART-2

ELECTRICAL MEASUREMENTS

Wheatstonebridge:- It is a DC bridge we find resistance.

→ In Wheatstone bridge the value of unknown resistance is determined by comparing with the known resistances.

→ product of opposite resistors are Equal.

→ For any bridge, the bridge should be under balenced condition.

→ current through galvanometer is '0'.

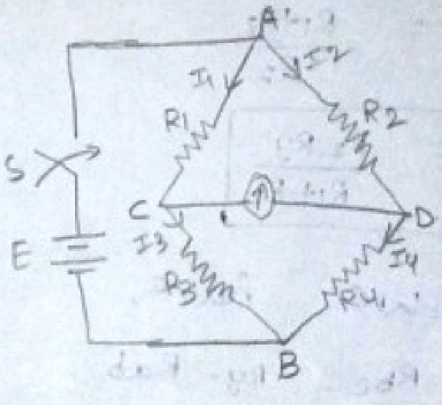
→ C & D potentials are at same so, ($V_C = V_D$)

→ In the taken resistor, any one of the resistor should be variable resistor. By using that resistor, we make the bridge balenced condition

potential at C = potential at D.

→ when the potential difference b/w the opposite terminals are Equal then the bridge is said to be under balenced condition.

from figure,



$V_{ac} = V_{ab}$ $\quad V_{ad} = V_{bd}$

$I_1 R_1 = I_3 R_3$ — ① $\quad I_2 R_2 = I_4 R_4$ — ②

In balenced condition no current flows through the galvanometer. so,

$$I_1 = I_2 \;,\; I_3 = I_4$$

$$\frac{①}{②} = \frac{I_1 R_1}{I_3 R_3} = \frac{I_2 R_2}{I_4 R_4}$$

$$\frac{I_2 R_1}{I_3 R_3} = \frac{I_2 R_2}{I_4 R_4}$$

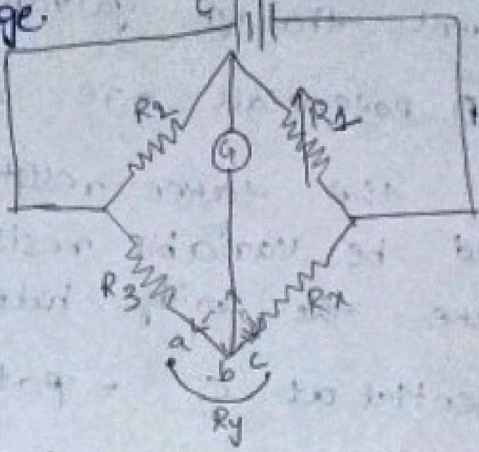
$$(\cancel{I_2}R_1)(\cancel{I_3}R_4) = (\cancel{I_2}R_2)(\cancel{I_3}R_3)$$

$$R_1 R_4 = R_2 R_3$$

$$\boxed{R_4 = \frac{R_2 R_3}{R_1}}$$

~~Kel~~

Kelvin bridge:- It is also DC bridge.
Here, also we find the value of Resistance when bridge is under balenced condition, it has same potential and voltage.



WKT:- condition:-

$$\frac{R_{bc}}{R_{ab}} = \frac{R_1}{R_2}$$

$$\frac{R_{bc}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{bc}+R_{ab}}{R_{ab}} = \frac{R_1+R_2}{R_2}$$

(from fig:- $R_{bc} + R_{ab} = R_y$)

$$\frac{R_y}{R_{ab}} = \frac{R_1+R_2}{R_2}$$

$$\boxed{R_{ab} = \frac{R_2 R_y}{R_1+R_2}}$$

Here, we have,

$$R_{bc} = R_y - R_{ab}$$

$$= R_y - \frac{R_2 R_y}{R_1+R_2}$$

$$= \frac{R_1 R_y + \cancel{R_2 R_y} - \cancel{R_2 R_y}}{R_1+R_2}$$

$$\boxed{R_{bc} = \frac{R_1 R_y}{R_1+R_2}}$$

from wheatstone bridge condition,

$$R_2(R_x+R_{bc}) = R_1(R_3+R_{ab})$$

$$R_x+R_{bc} = \frac{R_1}{R_2}\cdot(R_3+R_{ab})$$

$$R_x = \frac{R_1R_3}{R_2} + \frac{R_1R_{ab}}{R_2} - R_{bc}$$

(sub R_{ab}, R_{bc} values)

$$R_x = \frac{R_1R_3}{R_2} + \frac{\cancel{R_1 R_2 R_y}}{\cancel{R_2(R_1+R_2)}} - \frac{\cancel{R_1R_y}}{\cancel{R_1+R_2}}$$

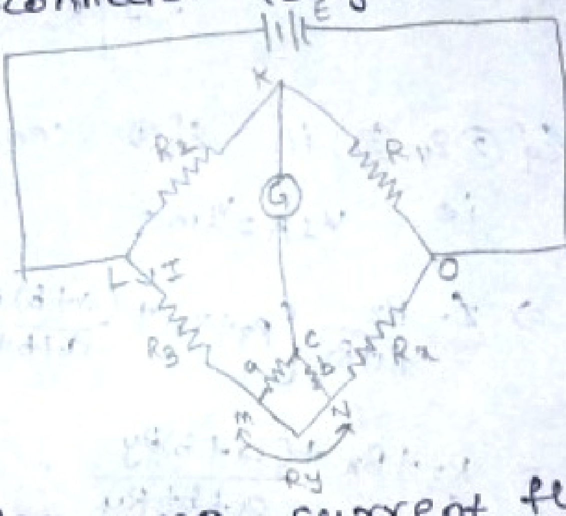
$$\boxed{R_x = \frac{R_1R_3}{R_2}}$$

Kelvin's double bridge:- It is also DC bridge

→ It is a modification of wheatstone's bridge

→ It is used to measure low resistance.

→ It uses a set of Ratio arms (a, b) they name double bridge they are connected to galvanometer through a point 'c'.



w.k.T

$$\frac{R_2}{R_1} = \frac{a}{b}$$

$$bR_2 = aR_1$$

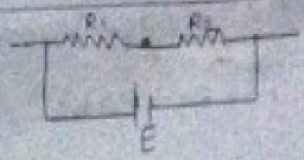
$$\boxed{\frac{R_1}{R_2} = \frac{b}{a}}$$

w.k.T

under balenced condition no current flows through galvanometer

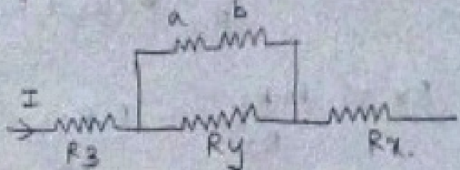
voltage drop(E), $V_{lK} = V_{lmc}$ → ①

first by applying voltage divider rule we find the value of V_{lK}



$V_{ek} = E \times \frac{R_2}{R_1+R_2}$

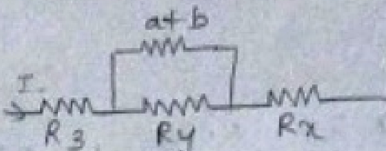
To find the value of E,



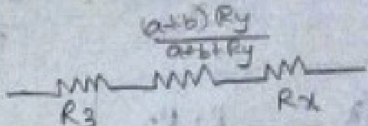
$V = IR$

$E = IR$

$E = I \cdot \left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right]$

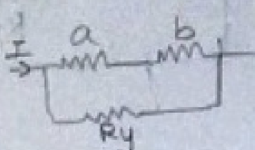


Substituting 'E' value.

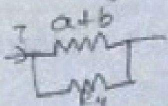


$$V_{ek} = I \cdot \frac{R_2}{R_1+R_2}\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] \quad -(2)$$

finding the value of $V_{emc} = V_{em} + V_{mc}$.



$\cancel{E = IR_3 + I\left[\frac{aR_y}{a+b+R_y}\right]}$



$= IR_3 + I\frac{a}{a+b}\left[\frac{(a+b)R_y}{a+b+R_y}\right]$

$\Rightarrow I\frac{(a+b)R_y}{a+b+R_y}$

$= IR_3 + I\frac{aR_y}{a+b+R_y}$

$= I\left[R_3 + \frac{aR_y}{a+b+R_y}\right] \quad -(3)$

sub Eq̂ (2) & (3) in Eq̂ (1) condition,

ie' $V_{ek} = V_{emc}$

$\Rightarrow \cancel{I} \cdot \frac{R_2}{R_1+R_2}\left[R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y}\right] = \cancel{I}\left[R_3 + \frac{aR_y}{a+b+R_y}\right]$

$= R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[R_3 + \frac{aR_y}{a+b+R_y}\right]\left[\frac{R_1+R_2}{R_2}\right]$

$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[R_3 + \frac{aR_y}{a+b+R_y}\right]\left[\frac{R_1}{R_2} + \cancel{\frac{R_2}{R_2}}\right]$

$R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[R_3 + \frac{aR_y}{a+b+R_y}\right]\left[\frac{R_1}{R_2} + 1\right]$

$$\cancel{R_3} + R_x + \frac{(a+b)R_y}{a+b+R_y} = \cancel{R_3} + \frac{R_1R_3}{R_2} + \frac{aR_y}{a+b+R_y} + \frac{R_1}{R_2}\left[\frac{aR_y}{a+b+R_y}\right]$$

$$R_x = \frac{R_1R_3}{R_2} + \left[\frac{aR_y}{a+b+R_y}\right]\frac{R_1}{R_2} + \frac{aR_y}{a+b+R_y} - \frac{(a+b)R_y}{a+b+R_y}$$

$$R_x = \frac{R_1R_3}{R_2} + \frac{R_1}{R_2}\left[\frac{aR_y}{a+b+R_y}\right] + \frac{\cancel{aR_y} - \cancel{aR_y} - bR_y}{a+b+R_y}$$

$$R_x = \frac{R_1R_3}{R_2} + \frac{R_1}{R_2}\left[\frac{aR_y}{a+b+R_y}\right] - \frac{bR_y}{a+b+R_y}$$

$$R_x = \frac{R_1R_3}{R_2} + \frac{aR_y}{a+b+R_y}\left[\frac{R_1}{R_2} - \frac{b}{a}\right] \quad \left[\because a \text{ is multiply \& divided}\right]$$

$$\left[\because \frac{R_1}{R_2} = \frac{b}{a}\right]$$

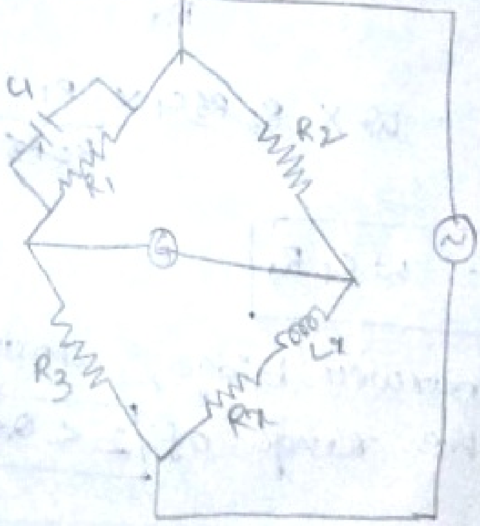
$$R_x = \frac{R_1R_3}{R_2} + \frac{aR_y}{a+b+R_y}\left[\cancel{\frac{R_1}{R_2} - \frac{R_1}{R_2}}\right]^{0}$$

$$\boxed{R_x = \frac{R_1R_3}{R_2}}$$

Maxwell Bridge:- It is an AC bridge.

→ In this bridge we measure an unknown Inductance by comparing with a known standard inductance.

we know under balenced condition, product of opposite impedances are Equal.



$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1} \quad -(1)$$

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_x = R_x + j\omega L_x$$

→ the capacitance is connected in Parallel with the resistance, the voltage drop across the resistance and capacitor is Equal.

Substituting the values in Eq(1)

$$Z_x = Z_2 Z_3 \cdot \frac{1}{Z_1}$$

$$R_x + j\omega L_x = R_2 R_3 \left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + R_2 R_3 C_1 j\omega.$$

Separating Real terms we get

$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

Separating imaginary terms we get

$$\boxed{L_x = R_2 R_3 C_1}$$

where the resistances are Expressed in ohms, inductance in henrys and capacitance in farads

Quality factor:- Quality factor which is represented in (Q).

$$Q = \omega \times \frac{L_x}{R_x}.$$

$$Q = \omega \times \frac{R_2 R_3 C_1}{\frac{R_2 R_3}{R_1}}$$

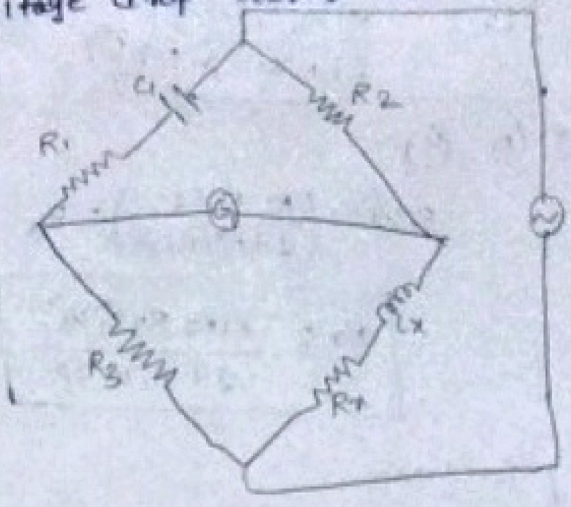
$$Q = \omega \times \cancel{R_2 R_3} C_1 \times \frac{R_1}{\cancel{R_2 R_3}}$$

$$\boxed{Q = \omega C_1 R_1}$$

In maxwell bridge, Inductance of a coil having 'Q' value in the range of $\boxed{1 < Q < 10}$

Hay's Bridge:- It is also an AC bridge. It is the modification of maxwell bridge

→ In Hay's bridge, the capacitor is connected in series with the resistance, the voltage drop across the capacitance and resistance are varied.



$$z_1 z_x = z_2 z_3 \quad \text{—①}$$

$$z_1 = R_1 - \frac{j}{\omega C_1}$$

$$z_2 = R_2$$

$$z_3 = R_3$$

$$z_x = R_x + j\omega L_x$$

$$\left(R_1 - \frac{j}{\omega C_1}\right)(R_x + j\omega L_x) = R_2 R_3$$

$$R_1 R_x + R_1 j\omega L_x - \frac{jR_x}{\omega C_1} - \frac{j^2 \omega L_x}{\omega C_1} = R_2 R_3$$

$$(\because j^2 = -1)$$

$$R_1 R_x + R_1 j\omega L_x - \frac{jR_x}{\omega C_1} + \frac{L_x}{C_1} = R_2 R_3$$

$$R_1 R_x + \frac{L_x}{C_1} + j\left(R_1 \omega L_x - \frac{R_x}{\omega C_1}\right) = R_2 R_3$$

Comparing Real parts:-

$$R_1 R_x + \frac{L_x}{C_1} = R_2 R_3 \quad \text{—②} \Rightarrow \frac{R_1 R_x C_1 + L_x}{C_1} = R_2 R_3$$

Comparing Imaginary parts:-

$$R_1 \omega L_x - \frac{R_x}{\omega C_1} = 0$$

$$R_1 \omega L_x = \frac{R_x}{\omega C_1} \quad \text{—③}$$

$$R_1 \omega^2 L_x C_1 = R_x \quad \text{—④}$$

Sub Eq④ in Eq②

$$\frac{R_1 (R_1 \omega^2 L_x C_1) C_1 + L_x}{C_1} = R_2 R_3$$

$$\frac{R_1^2\omega^2C_1^2L_x + L_x}{C_1} = R_2R_3$$

$$L_x(R_1^2\omega^2C_1^2+1) = R_2R_3C_1$$

$$\boxed{L_x = \frac{R_2R_3C_1}{1+R_1^2\omega^2C_1^2}} \quad (5)$$

Sub eq (5) in (4)

$$R_1\omega^2C_1\left(\frac{R_2R_3C_1}{1+R_1^2\omega^2C_1^2}\right) = R_x$$

$$\boxed{R_x = \frac{R_1R_2R_3C_1^2\omega^2}{1+R_1^2\omega^2C_1^2}} \quad (6)$$

Quality factor

$$Q = \omega \times \frac{L_x}{R_x}$$

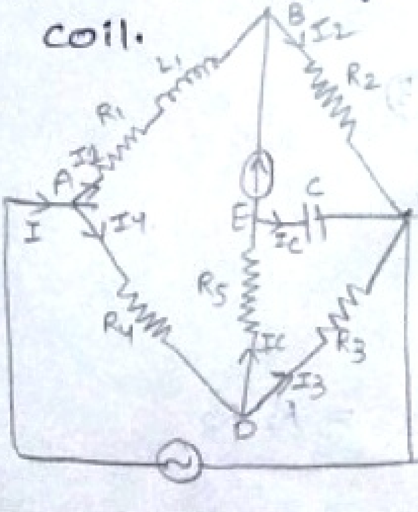
$$= \omega \times \frac{R_2R_3C_1}{\cancel{1+R_1^2C_1^2\omega^2}} \times \frac{\cancel{1+R_1^2\omega^2C_1^2}}{R_1R_2R_3C_1^2\omega^2}$$

$$= \boxed{Q = \frac{1}{R_1C_1\omega}}$$

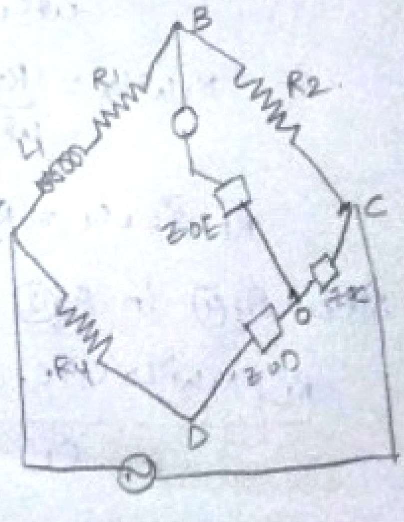
In hay's Bridge 'Q' value is in the range $\boxed{Q > 10}$

Anderson Bridge:- It is used to measure the inductance. It is AC bridge.

→ A large number of AC bridges are available and Anderson's bridge is an AC bridge used to measure self inductance of the coil.



⇒



At when the bridge is at balenced condition

$V_B = V_E$,

$V_{AB} = V_{ADE}$ & $V_{CB} = V_{CE}$ & $V_{DC} = V_{DCE}$

first,

$V_{CB} = V_{CE}$ from fig we can write

$I_2 R_2 = I_C \cdot \frac{1}{j\omega C}$

$I_C = I_2 R_2 j\omega C$ — ①

$V_{AB} = V_{ADE}$

$I_1 [R_1 + j\omega L_1] = I_4 R_4 + I_C R_5$

since at balenced conditions, $I_1 = I_2$

from Eq ① we can write,

$I_2 [R_1 + j\omega L_1] = I_4 R_4 + I_2 R_2 j\omega C R_5$

$I_4 R_4 = I_2 [R_1 + j\omega L_1 - j\omega C R_2 R_5]$ — ②

$V_{DC} = V_{DEC}$

$V_{DC} = V_{DE} + V_{EC}$

$R_3 (I_4 - I_C) = I_C \left[R_5 + \frac{1}{j\omega C}\right]$

$I_4 R_3 = I_C \left[R_5 + R_3 + \frac{1}{j\omega C}\right]$

from ① ($I_C = I_2 R_2 j\omega C$)

$$I_4 = \frac{I_2 R_2 j\omega C R_2}{R_3} \left[R_5 + R_3 + \frac{1}{j\omega C}\right] \text{ — ③}$$

sub Eq ③ in ②

$$\Rightarrow \frac{j\omega C \cancel{I_2} R_2 R_4}{R_3} \left[R_5 + R_3 + \frac{1}{j\omega C}\right] = \cancel{I_2} \left[R_1 + j\omega L_1 - j\omega C R_2 R_5\right]$$

$$\Rightarrow \frac{j\omega C R_2 R_4}{R_3} \left[\frac{R_5 j\omega C + R_3 j\omega C + 1}{j\omega C}\right] = R_1 + j\omega L_1 - j\omega C R_2 R_5$$

$$\Rightarrow R_5 j\omega C + R_3 j\omega C + 1 = (R_1 + j\omega L_1 - j\omega C R_2 R_5) \frac{R_3}{R_2 R_4}$$

$$\Rightarrow R_5 j\omega C + R_3 j\omega C + 1 = \frac{R_1 R_3}{R_2 R_4} + \frac{R_3 j\omega L_1}{R_2 R_4} - \frac{j\omega C \cancel{R_2} R_3 R_5}{\cancel{R_2} R_4}$$

comparing Real parts.

$$1 = \frac{R_1 R_3}{R_2 R_4}$$

$$R_2 R_4 = R_1 R_3$$

$$\boxed{R_1 = \frac{R_2 R_4}{R_3}}$$

comparing imaginary parts,

$$j\omega C(R_3+R_5) = \frac{j\omega L_1 R_3}{R_2 R_4} - \frac{j\omega C R_3 R_5}{R_4}$$

$$\cancel{j\omega C}(R_3+R_5) = \cancel{j\omega C}\left(\frac{L_1 R_3}{R_2 R_4 C} - \frac{R_3 R_5}{R_4}\right)$$

$$R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5 = \frac{L_1 R_3}{C}$$

$$L_1 = \frac{C}{R_3}(R_2 R_3 R_4 + R_2 R_4 R_5 + R_2 R_3 R_5)$$

$$L_1 = C\left(R_2 R_4 + \frac{R_2 R_4 R_5}{R_3} + R_2 R_5\right)$$

$$\boxed{L_1 = C R_2\left(R_4 + \frac{R_4 R_5}{R_3} + R_5\right)}$$

Schering Bridge:- It is one of the most important in ac bridge. It is used extensively for the measurement of capacitors.

→ The Schering bridge works on the principle of balancing the load on its arm.

→ the arm contains a parallel combinations of a resistor and a capacitor. and the standard arm contains only a capacitor.

→ As we know,

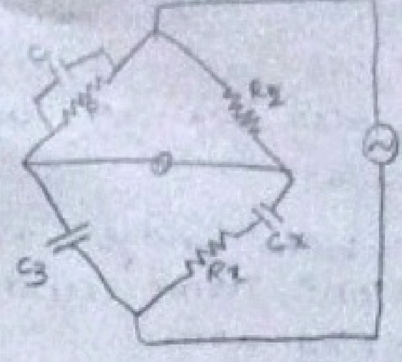
$$Z_1 Z_x = Z_2 Z_3$$

$$Z_x = \frac{Z_2 Z_3}{Z_1}$$

$$\frac{1}{Z_1} = \frac{1}{R_1} + j\omega C_1$$

$$Z_2 = R_2, \quad Z_3 = -\frac{j}{\omega C_3}$$

$$Z_x = R_x - \frac{j}{\omega C_x}.$$



$$R_x - \frac{j}{\omega C_x} = (R_2)\left(-\frac{j}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_x} = \left(-\frac{jR_2}{\omega C_3}\right)\left(\frac{1}{R_1} + j\omega C_1\right)$$

$$R_x - \frac{j}{\omega C_x} = -\frac{jR_2}{\omega C_3 R_1} - \frac{j^2 R_2 \cancel{\omega} C_1 \cancel{R_1}}{\cancel{\omega} C_3 \cancel{R_1}} \quad (j^2 = -1)$$

$$R_x - \frac{j}{\omega C_x} = \frac{-jR_2}{\omega C_3 R_1} + \frac{R_2 C_1}{C_3}.$$

comparing real parts

$$\boxed{R_x = \frac{R_2 C_1}{C_3}}$$

comparing Imaginary parts

$$\frac{\cancel{-j}}{\omega C_x} = +\cancel{j}\frac{R_2}{\omega C_3 R_1}$$

$$\frac{1}{\cancel{\omega} C_x} = \frac{R_2}{\cancel{\omega} C_3 R_1}$$

$$\boxed{C_x = \frac{R_1 C_3}{R_2}.}$$

quality factor $(Q) = \frac{\omega}{R_x C_x}$

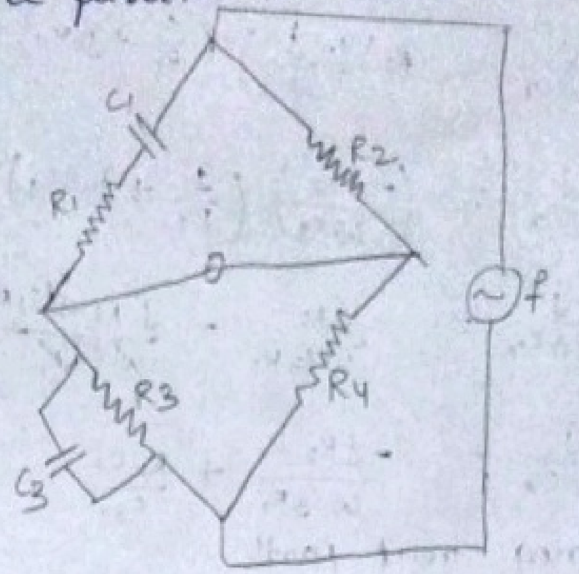
$$= \frac{\omega}{\left(\frac{R_2 C_1}{C_3}\right)\left(\frac{R_1 C_3}{R_2}\right)} \Rightarrow \frac{\omega \cancel{R_2} \cancel{C_3}}{\cancel{R_2} R_1 C_1 \cancel{C_3}}$$

$$\Rightarrow \frac{\omega}{R_1 C_1}$$

$$\boxed{Q = \frac{\omega}{R_1 C_1}}$$

Wein bridge:- The wein bridge is presented here not only for its uses as an ac bridge to measure frequency, but also for its application in various other useful circuits.

→ In wein bridge it has a series RC combination in one arm and a parallel RC combination in the adjoining arm.



$$Z_1 = R_1 - \frac{j}{\omega C_1}$$

$$Z_2 = R_2$$

$$\frac{1}{Z_3} = \frac{1}{R_3} + j\omega C_3$$

$$Z_4 = R_4.$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$Z_2 = \frac{Z_1 Z_4}{Z_3}.$$

$$R_2 = \left(R_1 - \frac{j}{\omega C_1}\right)(R_4) \cdot \left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \left(\frac{R_1 \omega C_1 - j}{\omega C_1}\right)(R_4)\left(\frac{1}{R_3} + j\omega C_3\right)$$

$$R_2 = \left(R_4 R_1 - \frac{R_4 j}{\omega C_1}\right)\left(\frac{1}{R_3} + j\omega C_3\right) \Rightarrow R_2 = \frac{R_4 R_1}{R_3} - \frac{R_4 j}{\omega C_1 R_3} + j\omega C_3 R_1 R_4 - \frac{R_4 C_3}{C_1}$$

comparing real parts and imaginary parts :

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{R_4 C_3}{C_1}$$

$$\boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}}$$

$$\frac{R_4 j}{\omega C_1 R_3} = j\omega C_3 R_1 R_4$$

$$\omega^2 = \frac{R_4}{C_3 R_1 \cancel{R_4} C_1 R_3} \Rightarrow \omega = \frac{1}{\sqrt{R_1 R_3 C_1 C_3}} \Rightarrow 2\pi f = \frac{1}{\sqrt{R_1 R_3 C_1 C_3}}$$

$$f = \frac{1}{2\pi\sqrt{R_1 R_3 C_1 C_3}} \quad \left(\because \begin{array}{l} R_1 = R_3 = R \\ C_1 = C_3 = R \end{array}\right)$$

$$f = \frac{1}{2\pi\sqrt{R^2 C^2}} \Rightarrow \boxed{f = \frac{1}{2\pi\sqrt{RC}}}$$

$$\boxed{f = \frac{1}{2\pi RC}}$$

Q-METER:-

→ The Q-meter is an instrument designed to measure the electrical properties of coils and capacitors. Some of the

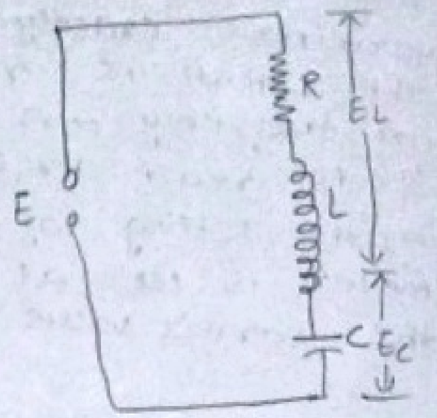
→ The voltage and current relationships of a series-resonant circuit. At resonance,

$$X_L = X_C.$$

$$E_C = IX_C = IX_L$$

$$E = I[R + X_C + X_C] \quad (\because X_C = X_L)$$

$$\boxed{E = IR}$$



where, E = applied voltage.

I = circuit current

E_C = voltage across the capacitor

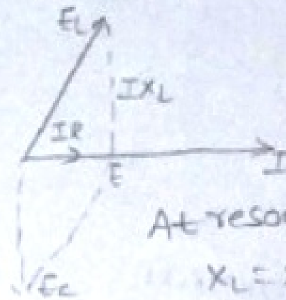
X_C = capacitive reactance

X_L = inductive reactance

R = coil resistance.

The magnification of the circuit, by definition is Q, where,

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{E_C}{E}$$



At resonance:

$X_L = X_C$

$I = \frac{E}{R}$

$$Q = \frac{IX_C}{IR} = \frac{E_C}{E}$$

$$Q = \frac{E_C}{E}$$

$$\boxed{E_C = QE}$$

$$\boxed{E_C \propto Q}$$

→ The wide-range oscillator with a frequency range from 50KHz to 50MHz delivers current to a low-value shunt resistance R_{SH}.

→ The inductance of the coil can be calculated from the known values of frequency (f) and resonating capacitance (C).

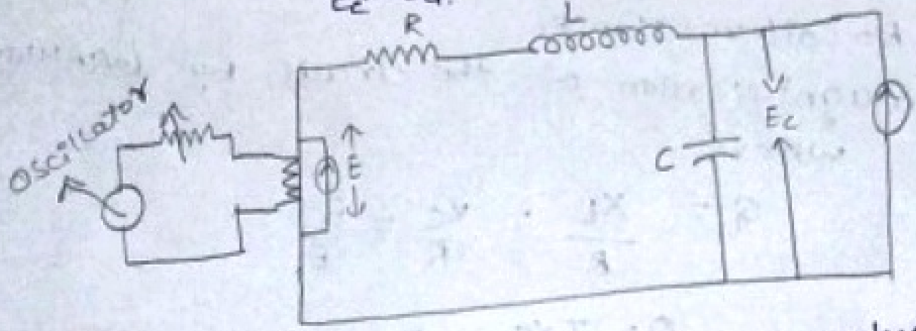
Since, $X_L = X_C$ and $L = \frac{1}{(2\pi f)^2 C}$ henry.

There are three methods for connecting unknown components to the test terminals of Q meter:

1. Direct method
2. Series method
3. Parallel method.

Direct method:- In this method the Q-factor of Inductor (or) capacitor measured directly. $\boxed{E_C \propto Q}$
Most coils can be connected directly across the test terminals, exactly as shown in the basic Q-circuit. The circuit is resonated by adjusting either the oscillator frequency (or) the resonating capacitor. The indicated 'Q' is read directly from the 'circuit Q" meter, modified by the setting of the "multiply Q by" meter when the last meter is set at unity mark, the "circuit Q" meter reads the correct value of Q directly.

$$E_C \propto Q$$
$$E_c = EQ.$$



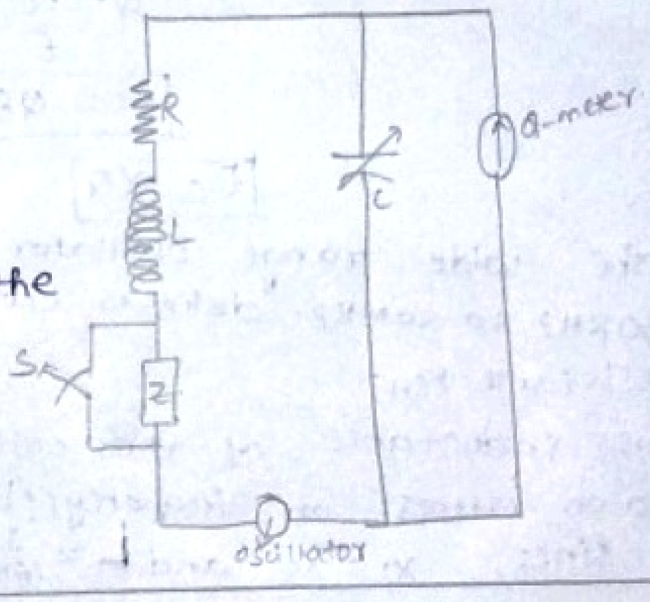
Series method:-
→ low-impedance components, such as low-value resistors, small coils and large capacitors are measured in series with the measuring circuit

for the reference condition,

$$X_{C_1} = X_L \text{ (or) } \frac{1}{\omega C_1} = \omega L \quad \text{--- (1)}$$

and neglecting the resistance of the measuring circuit

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}.$$



$X_L + X_S = X_{C_2}$ —②

$X_S = X_{C_2} - X_L$

from Eq① $\Rightarrow X_S = X_{C_2} - X_{C_1}$

$X_S = \frac{1}{\omega C_2} - \frac{1}{\omega C_1}$

$X_S = \frac{\omega C_1 - \omega C_2}{\omega^2 C_1 C_2} \Rightarrow X_S = \frac{\cancel{\omega}(C_1 - C_2)}{\omega^2 C_1 C_2}$

$$\boxed{X_S = \frac{C_1 - C_2}{\omega C_1 C_2}}$$

If 'Z' Inductor,

$\omega L_S = \frac{C_1 - C_2}{\omega C_1 C_2}$

$$\boxed{L_S = \frac{C_1 - C_2}{\omega^2 C_1 C_2}}$$

If 'Z' is capacitor

$\frac{1}{\cancel{\omega} C_S} = \frac{C_1 - C_2}{\cancel{\omega} C_1 C_2}$

$\frac{1}{C_S} = \frac{C_1 - C_2}{C_1 C_2}$

$$\boxed{C_S = \frac{C_1 C_2}{C_1 - C_2}}$$

$\rightarrow X_S$ is inductive if $C_1 > C_2$ and capacitive if $C_1 < C_2$.

To determine the quality factor.

$Q_S = \frac{X_S}{R_S}$

$(R = X/Q)$

$R_S = R_{C_2} - R_{C_1}$

$= \frac{X_{C_2}}{Q_2} - \frac{X_{C_1}}{Q_1}$

$= \frac{1}{\omega C_2 Q_2} - \frac{1}{\omega C_1 Q_1}$

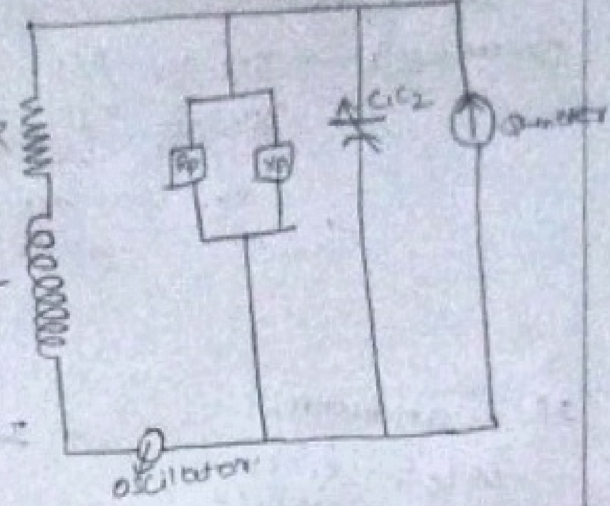
$= \frac{\omega C_1 Q_1 - \omega C_2 Q_2}{\omega^2 C_1 C_2 Q_1 Q_2} \Rightarrow \frac{\cancel{\omega}(C_1 Q_1 - C_2 Q_2)}{\omega^2 C_1 C_2 Q_1 Q_2}$

$$\boxed{R_S = \frac{C_1 Q_1 - C_2 Q_2}{\omega C_1 C_2 Q_1 Q_2}}$$

$Q_S = \frac{X_S}{R_S} \Rightarrow \frac{C_1 - C_2}{\cancel{\omega C_1 C_2}} \times \frac{\cancel{\omega C_1 C_2} Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}$

$$\boxed{Q_S = \frac{Q_1 Q_2 (C_1 - C_2)}{C_1 Q_1 - C_2 Q_2}} \quad \text{(or)} \quad \boxed{Q_S = \frac{Q_1 Q_2 \Delta C}{C_1 Q_1 - C_2 Q_2}}$$

parallel method:- High-impedance components, such as high-value resistors, certain inductors, and small capacitors, are measured by connecting them in parallel with the measuring circuit.



→ In a parallel circuit, computation of the unknown impedance is best approached in terms of its parallel components Xp and Rp.

$$X_L = X_{C_1}$$

$$\omega L = \frac{1}{\omega C_1}$$

$$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

when the unknown impedance is not connected into the circuit and the capacitor is tuned for resonance.

$$X_L = X_p \parallel X_{C_2}$$

$$\frac{1}{X_{C_1}} = \frac{1}{X_p} + \frac{1}{X_{C_2}} \Rightarrow \omega C_1 = \frac{1}{X_p} + \omega C_2$$

$$\boxed{X_p = \frac{1}{\omega(C_1 - C_2)}}$$

If $X_p = \omega L_p$. If unknown is inductive

$$\omega L_p = \frac{1}{\omega(C_1 - C_2)} \Rightarrow \boxed{L_p = \frac{1}{\omega^2(C_1 - C_2)}}$$

If $X_p = \frac{1}{\omega C_p}$ if unknown is capacitive

$$\frac{1}{\omega C_p} = \frac{1}{\omega(C_1 - C_2)}$$

$$\boxed{C_p = C_1 - C_2}$$

In a parallel resonant circuit the total resistance at resonance is equal to the product of the circuit Q and the reactance of the coil.

$$R_T = Q_2 X_L$$

$$R_T = Q_2 X_{C_1}$$

$$\boxed{R_T = \frac{Q_2}{\omega C_1}}$$

The resistance of the unknown impedance is most easily found by computing the conductances in the circuit.

G_T = total conductance of the resonant circuit

G_p = conductance of the unknown impedance

G_L = conductance of the working coil.

$$G_T = G_p + G_L \quad (\text{or}) \quad G_p = G_T - G_L$$

$$G_T = \frac{1}{R_T} = \frac{\omega C_1}{Q_2}$$

$$G_L = \frac{1}{R+j\omega L} \times \frac{R-j\omega L}{R-j\omega L}$$

$$G_L = \frac{R-j\omega L}{R^2+\omega^2L^2}$$

$$G_L = \frac{R}{R^2+\omega^2L^2} - \cancel{\frac{j\omega L}{R^2+\omega^2L^2}}^{\text{neglected}}$$

($\because$ Due to having of imaginary parts. it has been neglected)

$$G_L = \frac{R}{R^2+\omega^2L^2}$$

$$G_p = G_T - G_L$$

$$= \frac{\omega C_1}{Q_2} - \frac{R}{R^2+\omega^2L^2}$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R}\left[\frac{1}{1+\left(\frac{\omega^2L^2}{R^2}\right)}\right]$$

$$\left(\because Q_1 = \frac{X_L}{R} = \frac{\omega L}{R}\right)$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R}\left[\frac{1}{1+(Q_1)^2}\right]$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R}\cdot\frac{1}{Q_1^2}$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{R}\cdot\frac{1}{Q_1}\cdot\frac{1}{Q_1}$$

$$= \frac{\omega C_1}{Q_2} - \frac{1}{\cancel{R}Q_1}\cdot\omega C_1\cancel{R}$$

$$= \frac{\omega C_1}{Q_2} - \frac{\omega C_1}{Q_1} \Rightarrow \omega C_1\left(\frac{1}{Q_2} - \frac{1}{Q_1}\right)$$

$$G_p = \omega C_1\left(\frac{1}{Q_2} - \frac{1}{Q_1}\right)$$

$$\frac{1}{R_p} = G_p = \omega C_1\left(\frac{Q_1 - Q_2}{Q_1 Q_2}\right)$$

The 'Q' of the unknown is then found by using

$$Q_p = \frac{R_p}{X_p}$$

$$= \frac{Q_1 Q_2}{\cancel{\omega} C_1(Q_1 - Q_2)} \times \cancel{\omega}(C_1 - C_2)$$

$$\boxed{Q_p = \frac{(C_1 - C_2)\, Q_1 Q_2}{C_1 (Q_1 - Q_2)}}$$

(or)

$$\boxed{Q_p = \frac{(C_1 - C_2)\, Q_1 Q_2}{C_1\, \Delta Q}}$$