R20

UNIT – II

LINEAR WIRE ANTENNA

* Retarded Potentials
* Radiation from small electric Dipole
* Quarter wave Monopole and Half wave Dipole – Current distributions
* Evaluation of field components
* Power radiated
* Radiation resistance
* Directivity
* Introduction to Antenna Theorems
* Loop antennas.

Potential functions and Electromagnetic fields

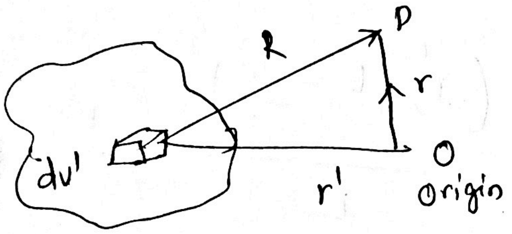
(Retarded potentials)

→ for obtaining the potentials for the electromagnetic field there are different approaches. In the first approach, using trial and error, the potential for the electric and magnetic fields are generalized. These potentials satisfy the Maxwell's equations. This approach is called heuristic approach.

The second approach is to start with the Maxwell's equation and then derive the differential equations that the potentials satisfy.

The third approach is to obtain directly the solutions the derived differential equations for the potentials.

Heuristic approach: —



Consider the uniform volume charge density ρ_v, over the given volume consider the differential volume dv' at point distance r' from the origin, where the charge density is $\rho_v(r')$.

The scalar electric potential V at point P can be expressed interms of a static charge distribution as,

$$V(r) = \int_V \frac{\rho_v(r')\,dv'}{4\pi\epsilon_0} \rightarrow ①$$

fundamental electric field is $\bar{E} = -\nabla V$

Vector magnetic potential $\bar{A}$ interms of current distribution

is $$\bar{A}(r) = \int_V \frac{\mu \bar{J}(r')}{4\pi R}\,dv' \rightarrow ③$$

Fundamental magnetic field is $\bar{B} = \nabla \times \bar{A}$

$$\mu\bar{H} = \nabla \times \bar{A}$$

equations 1&3 are the potentials for the static electric and magnetic ~~pot~~ fields, where the charge and current distribution donot vary with time.

But the charge and current distributions producing the electromagnetic field vary with the time.

$$V(r',t) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho_v\,(r',t)}{R}\,dv'$$

$$\bar{A}\,(r',t) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\,(r',t)}{R}\,dv'$$

Where $R = r - r'$

the equations w.r.t. time delay

$$V(r',t) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho_v\left(r', t - \frac{R}{v}\right)}{R}\,dv'$$

$$\bar{A}\,(r',t) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\left(r', t - \frac{R}{v}\right)}{R}\,dv'$$

These eqs are potentials are delayed or retarded by the time $\frac{R}{v}$.

(ii) Maxwell's Equations Approach:

for the time varying fields, Maxwell's equation in the point form are given by

$$\nabla\times\bar{E} = -\mu\frac{\partial\bar{H}}{\partial t} \quad -1$$

$$\nabla\times\bar{H} = \bar{J} + \varepsilon\frac{\partial\bar{E}}{\partial t} \quad -2$$

$$\nabla\cdot\bar{E} = \frac{\rho_V}{\varepsilon} \quad -3$$

$$\nabla\cdot\bar{H} = 0 \quad -4$$

From eq (4) it is clear that divergence is zero
So, Curl of gradient of scalar is vector is always zero

$$\mu \bar{H} = \nabla \times \bar{A} \quad -(5)$$

putting value of $\mu \bar{H}$ in eq (1)

$$\nabla \times \bar{E} = -\cancel{\mu} \frac{\partial}{\partial t}\left[\frac{\nabla \times \bar{A}}{\cancel{\mu}}\right]$$

Interchanging operator on R.H.S of above eq.

$$\nabla \times \bar{E} = -\left[\nabla \times \frac{\partial \bar{A}}{\partial t}\right]$$

$$\therefore \nabla \times \bar{E} + \nabla \times \frac{\partial \bar{A}}{\partial t} = 0$$

$$\therefore \nabla \times \left[\bar{E} + \frac{\partial \bar{A}}{\partial t}\right] = 0 \quad (6)$$

According to vector identity curl of a gradient of a scalar is always zero. So, eq (6) becomes $\bar{E} + \frac{\partial \bar{A}}{\partial t}$

$$\bar{E} + \frac{\partial \bar{A}}{\partial t} = -\nabla V \quad -(7)$$

Electric field strength $\bar{E} = -\nabla V - \frac{\partial \bar{A}}{\partial t} \quad -(8)$

from eq (5) & (8) $\bar{E}$ and $\bar{H}$ expressed in terms of scalar potential V and vector potential $\bar{A}$.

Eq (5) & (8) satisfy the maxwell's equations eq (1) & (4)

Substituting eq (8) & (5) in eq (2).

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \bar{A}\right) = \bar{J} + \varepsilon \frac{\partial}{\partial t}\left[-\nabla V - \frac{\partial \bar{A}}{\partial t}\right]$$

Interchanging the operators

$$\frac{1}{\mu}\left[\nabla \times \nabla \times \bar{A}\right] = \bar{J} + \varepsilon\left[-\nabla \frac{\partial V}{\partial t} - \frac{\partial^2 \bar{A}}{\partial t^2}\right]$$

$$\nabla \times \nabla \times \bar{A} = \mu \bar{J} - \varepsilon \mu \nabla \frac{\partial V}{\partial t} - \varepsilon \mu \frac{\partial^2 \bar{A}}{\partial t^2} \rightarrow (9)$$

from Vector Identity

$$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

putting eq 9 Using above identity

* operators are interchanging

$$\nabla^2 \bar{A} - \nabla(\nabla \cdot \bar{A}) = -\mu \bar{J} + \mu \varepsilon \nabla \frac{\partial V}{\partial t} + \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} \quad — (10)$$

subtituting $\bar{E}$ from eq (6) in eq (3)

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\varepsilon}$$

$$\nabla \cdot \left(-\nabla V - \frac{\partial \bar{A}}{\partial t} \right) = \frac{\rho_v}{\varepsilon}$$

* operators are interchanging.

$$\nabla^2 V + \nabla \cdot \frac{\partial \bar{A}}{\partial t} = -\frac{\rho_v}{\varepsilon} \quad — (11)$$

Eq 10 & 11 are differential equations both unknown $\bar{A}$ and V appear.

The divergence of $\bar{A}$ from eq 10

$$\nabla \cdot \bar{A} = -\mu \varepsilon \frac{\partial V}{\partial t} \rightarrow (12)$$

Substitute this in eq (10)

$$\nabla^2 \bar{A} - \nabla\left(-\mu \varepsilon \frac{\partial V}{\partial t}\right) = -\mu \bar{J} + \mu \varepsilon \nabla \frac{\partial V}{\partial t} + \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\nabla^2 \bar{A} + \mu \varepsilon \cancel{\nabla \frac{\partial V}{\partial t}} = -\mu \bar{J} + \cancel{\mu \varepsilon \nabla \frac{\partial V}{\partial t}} + \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2}$$

$$\therefore \nabla^2 \bar{A} - \mu \varepsilon \frac{\partial^2 \bar{A}}{\partial t^2} = -\mu \bar{J} \rightarrow (13)$$

Similarly Substitute $\nabla \cdot \bar{A}$ in eq (11)

$$\nabla^2 V + \nabla \cdot \frac{\partial \bar{A}}{\partial t} = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 V + \frac{\partial}{\partial t}\left(-\mu \varepsilon \frac{\partial V}{\partial t}\right) = -\frac{\rho_v}{\varepsilon}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = \frac{\rho v}{\varepsilon} \quad \text{— (14)}$$

→ operator interchange.

The equations 13, 14 are standard wave equations including source terms.

$$A(r, t) = \frac{\mu}{4\pi} \int_V \frac{\bar{J}(r', t - R/v)}{R} \, dv'$$

$$V(r, t) = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho v (r', t - R/v)}{R} \, dv'$$

Radiation from Small electric Dipole: —

To calculate the electromagnetic field radiated in the Space by a short dipole, the retarded potential is used. A short dipole is an alternating current element. It is also called an oscillating current element. An alternating current element is considered as the basic source of radiation. It can be used as a building block for antenna analysis.

In general, a current element IdL nothing but an element of length dL; carrying filamentary current I. The length of this wire is assumed to be very short. So filamentary current can be considered as constant along the length of the element. In such cases, an antenna can be considered as made up of large numbers of such elements connected end to end.

Small dipole antenna is also called hertzian dipole. Hertzian dipole is nothing but an infinitesimal current element IdL. It consist two equal and opposite charges at the end of the current element separated by a short distance dL. is shown in the figure.

The wire between the two spheres where charges can accumulate is very thin as compared to the radius of the sphere. Thus current I is uniform through the wires. Also the distance dL is greater as compared to the radii of the sphere.

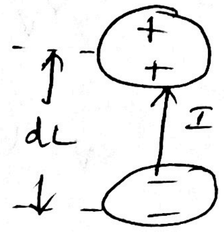


fig: Hertzian dipole

$$i = I \cos \omega t$$

Then the charge accumulated at the ends of the element and current flowing through the wire are related to each other by expression

$$dq = I \cos \omega t \, dt$$

Substituting the value of q in terms of current I we will get

$$E_\theta = \frac{I \, dL \sin\theta \sin \omega t'}{4 \pi \varepsilon \omega r^3}$$

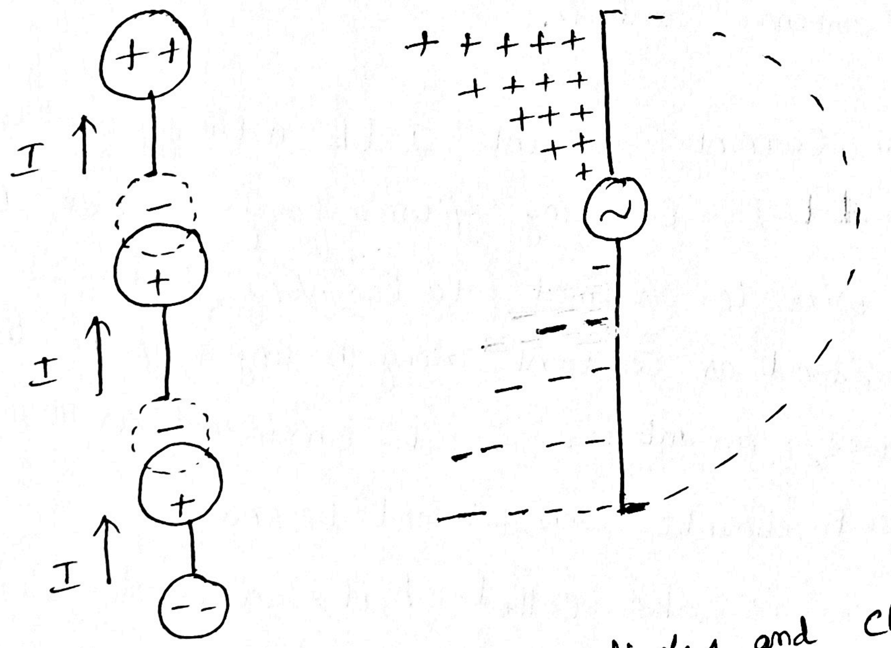


fig: Chain of hertzian dipoles and charge and current distribution on linear antenna.

When such Hertzian dipoles are connected end to end forming a practical antenna. It observed that positive charge at one end of the dipoles get cancelled by the equal and opposite lower charge end of the next dipole. Hence when the current is uniform along the antenna, then there is no charge accumulation at the ends of the dipole which indicates that $\frac{1}{r^3}$ term is absent and only induction and radiation fields are present.

But if the current through antenna is not uniform throughout then there is a accumulation of charge. These charges causes stronger electric field component normal to the surface of the wire.

$$P_r = \frac{\eta_0}{2}\left[\frac{\omega I dL \sin\theta}{4\pi r c}\right]^2$$

Current Distributions in Half Dipole and the Monopole. (5)

→ In order to calculate the radiated electromagnetic field of the longer antenna, the current distribution along the antenna must be known. The current distribution can be obtained by solving the Maxwell's equations for the time varying fields with the proper boundary conditions. But it is observed that the actual calculation of the current distribution of the cylindrical antenna is very difficult and complicated task. The mathematical expression obtained by solving the Maxwell's equations with appropriate boundary conditions are very complicated. Hence, in general it is a common practice to approximate the current distribution that is more or less same as the real distribution and from that approximate field expressions are calculated.

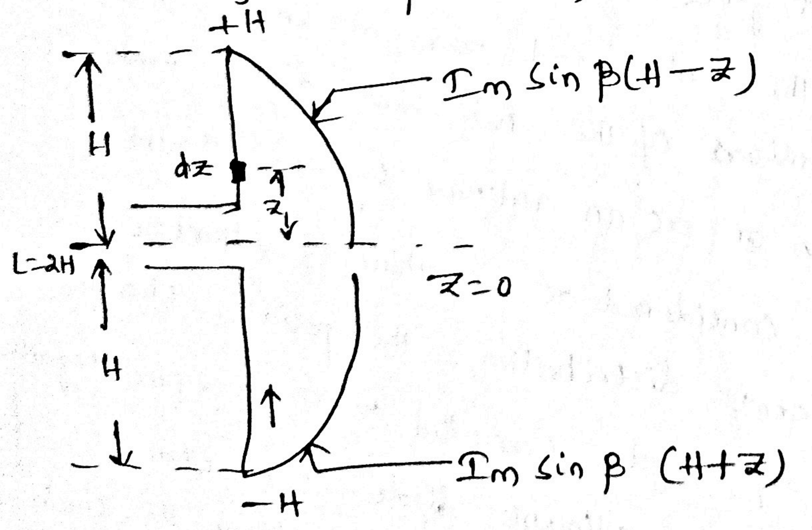
A very commonly used antenna is the half wave dipole with a length one half of the free space wave length of radiated wave. It is found that the linear current distribution is not suitable for thic antenna. But when such antenna is fed its centre with the help of a transmission line, it gives a current which is approximately sinusoidal, with max at the centre and zero at ends. The UHF, and VHF regions, the dimensions of the half wave dipole make it most suitable as an antenna or as an antenna system element.

The half wave dipole can be considered as a chain of Hertzian dipoles. For the uniform current distribution, the positive charges at the end of one Hertzian dipole gets cancelled with an equal negative charge at the opposite end of the adjacent dipole. But when the current distribution is not constant, the successive dipoles of the chain have slightly different current amplitudes, where adjacent charges are not cancelled completely.

Power radiated by the Halfwave Dipole and the Monopole.

→ A dipole antenna is a vertical radiator fed in the centre. It produces max. radiation in the plane normal to the axis. For such a dipole antenna, the length specified is the over all length. The vertical antenna of height $H = \frac{L}{2}$ produces radiation characteristics above the plane which is similar to that produced by the dipole antenna of length $L = 2H$. The vertical antenna is a monopole.

In general antenna requires large current to radiate large amount of power. To generate such large current at radio frequency, it is practically impossible. In case Hertzian dipole the expression for $\bar{E}$ and $\bar{H}$ are derived assuming uniform current throughout the length. But we have studied that at the ends of the antenna current is zero. In other words the current is not uniform through out the length as it is maximum at centre and zero at the ends. Hence practically Hertzian dipole is not used. The practically used antennas are halfwave dipole ($\lambda/2$) and quarter wave monopole ($\lambda/4$).



Monopole: Single rod / conductor

Dipole: Two conductor elements / rods

Fig: Assumed sinusoidal current distribution in halfwave dipole.

→ The halfwave dipole consists two legs each of length $\frac{L}{2}$. The physical length of half wave dipole at the frequency of operation is $\lambda/2$ in the free space.

→ The quaterwave monopole consists single vertical leg erected on the perfect ground i.e perfect conductor. The length of the quarter wave monopole is $\lambda/4$.

For the calculation of the electromagnetic fields, assumed sinusoidal current distribution along the halfwave dipole and quarter wave monopole are shown in the fig.

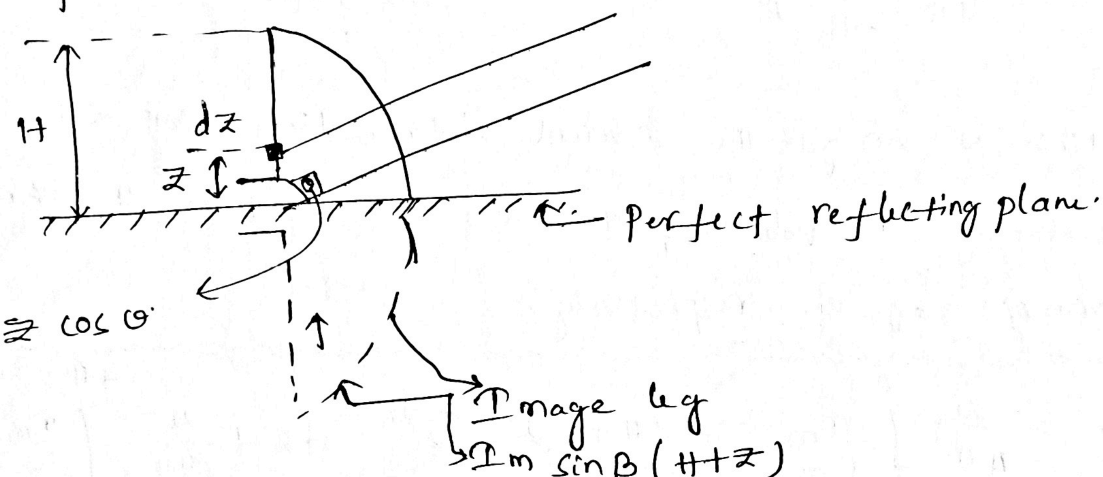


Fig: Assumed sinusoidal current distribution in quarter wave monopole.

consider the assumed sinusoidal current distribution in the quarter wave monopole and halfwave dipole. The current element I dz is placed at a distance z from z=0 plane.
Let Im be the max value of the current in the current element.
Then the sinusoidal current distribution is given by,

$$I = I_m \sin\beta(H-z) \text{ for } z>0 \quad \rightarrow ①$$

and

$$I = I_m \sin\beta(H+z) \text{ for } z<0. \quad \rightarrow ②$$

consider a point p located at a far distance from the current element I dz. Then the vector potential at a point P due to the current element I dz is given by

$$d \cdot A_z = \frac{\mu I}{4\pi R} e^{-j\beta R} \cdot dz \rightarrow ③$$

R is the distance of point P from the current element. The total vector potential at point P due to all such currents can be obtained by integrating the vector potential dA_z over the total length of the antenna.

$$\therefore A_z = \frac{\mu}{4\pi} \int_{-H}^{H} \frac{I}{R} e^{-j\beta R} \cdot dz$$

$$= \frac{\mu}{4\pi} \int_{-H}^{0} \frac{I}{R} e^{-j\beta R} dz + \frac{\mu}{4\pi} \int_{0}^{+H} \frac{1}{R} e^{-j\beta R} \cdot dz \rightarrow ④$$

from eq 1 & 2 we can get the current distributions for $z<0$, and $z>0$.

Thus substituting the values of I from eq. 2 & 1 in the first and second term of eq 4 respectively.

$$\therefore A_z = \frac{\mu}{4\pi} \int_{-H}^{0} \frac{I_m \sin\beta(H+z)}{R} e^{-j\beta R} dz + \frac{\mu}{4\pi} \int_{0}^{+H} \frac{I_m \sin\beta(H-z)}{R} \cdot e^{-j\beta R} \cdot dz \rightarrow ⑤$$

At this point we have to make certain assumptions, so as to calculate distant or radiation field. Assuming $R \approx r$, replace R in the denominators only by r.

So, $R = r - z\cos\theta$.

Replace R in the numerator terms by $(r - z\cos\theta)$. Then the eq.

$$A_z = \frac{\mu}{4\pi} \int_{-H}^{0} \frac{I_m \sin\beta(H+z)}{r} e^{-j\beta(r - z\cos\theta)} \cdot dz +$$

$$\frac{\mu}{4\pi} \int_{0}^{H} \frac{I_m \sin\beta(H-z)}{r} e^{-j\beta(r - z\cos\theta)} \cdot dz$$

$$\therefore A_z = \frac{\mu I_m}{4\pi r}\left[\int_{-H}^{0} \sin\beta(H+z)\, e^{-j\beta r}\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{+H} \sin\beta(H-z)\, e^{-j\beta r}\, e^{j\beta z\cos\theta}\, dz\right]$$

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0} \sin\beta(H+z)\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{+H} \sin\beta(H-z)\, e^{j\beta z\cos\theta}\, dz\right] \rightarrow (6)$$

for quarterwave monopole

$$H = \lambda/4 \text{ and } \beta = \frac{2\pi}{\lambda}$$

$$\therefore \sin\beta(H+z) = \sin(\beta H + \beta z) = \sin\left(\frac{\pi}{2} + \beta z\right)$$

$$\sin\beta(H-z) = \sin(\beta H - \beta z) = \sin\left(\frac{\pi}{2} - \beta z\right)$$

But $\sin\left(\frac{\pi}{2} + \beta z\right) = \sin\left(\frac{\pi}{2} - \beta z\right) = \cos\beta z \rightarrow \cancel{7}.$

Substituting values of sin terms in eq 6

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{-H}^{0} \cos\beta z\, e^{j\beta z\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

Now $\int_{-H}^{0} e^{+j\theta}\, d\theta = \int_{0}^{H} e^{-j\theta}\, d\theta$. Hence using this property, changing limits of integration

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H} \cos(-\beta z)\, e^{-j\beta z\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_{0}^{H} \cos\beta z\, e^{-j\beta\cos\theta}\, dz + \int_{0}^{H} \cos\beta z\, e^{j\beta z\cos\theta}\, dz\right]$$

$\because \cos(-\theta) = \cos\theta.$

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\int_0^H \cos\beta z\,(e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta}).dz\right] \quad \to (8)$$

By Euler's identity

$$e^{j\beta z\cos\theta} = \cos(\beta z\cos\theta) + j\sin(\beta z\cos\theta)$$

$$e^{-j\beta z\cos\theta} = \cos(\beta z\cos\theta) - j\sin(\beta z\cos\theta)$$

$$\therefore e^{j\beta z\cos\theta} + e^{-j\beta z\cos\theta} = 2\cos(\beta z\cos\theta) \quad \to (9)$$

Putting the value of the term inside the bracket in eq (8) we get

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H (\cos\beta z)\,2[\cos(\beta z\cos\theta)]\,dz$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H 2\cos\beta z\cos(\beta z\cos\theta)\,dz \quad \to (10)$$

from the trigonometric identity

$$2\cos A\cos B = \cos(A-B) + \cos(A+B)$$

Using this property in eq 10.

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^H \{\cos[\beta z + \beta z\cos\theta] + \cos[\beta z - \beta z\cos\theta]\}\,dz$$

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\int_0^{H=\lambda/4} \{\cos\beta z(1+\cos\theta) + \cos\beta z(1-\cos\theta)\}\,dz \quad \to (11)$$

Integrating w.r.t. z and putting value of $H = \lambda/4$

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{\sin\beta z(1+\cos\theta)}{\beta(1+\cos\theta)} + \frac{\sin\beta z(1-\cos\theta)}{\beta(1-\cos\theta)}\right]_0^{\lambda/4}$$

finding L.C.M

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r}\left[\frac{[\sin\beta z(1+\cos\theta)](1-\cos\theta)+[\sin\beta z(1-\cos\theta)](1+\cos\theta)}{\beta(1-\cos^2\theta)}\right]_0^{\lambda/4}$$

$$\therefore A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi r\beta}\left[\frac{(1-\cos\theta)\left[\sin\pi/2(1+\cos\theta)\right]+(1+\cos\theta)\left[\sin\pi/2(1-\cos\theta)\right]}{\sin^2\theta}\right] \rightarrow (12)$$

$\because \beta z = \frac{\pi}{2}$ and $(1-\cos^2\theta) = \sin^2\theta$.

Again using property

$$\sin(\pi/2+\theta) = \sin(\pi/2-\theta) = \cos\theta.$$

$$\therefore \sin(\pi/2+\pi/2\cos\theta) = \sin(\pi/2-\pi/2\cos\theta) = \cos(\pi/2\cos\theta) \rightarrow (13)$$

Substituting values of the sine-terms from eq 13 in eq. 12 we get

$$A_z = \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{(1-\cos\theta)\cos(\pi/2\cos\theta)+(1+\cos\theta)\cdot\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{2\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]$$

$$= \frac{\mu I_m e^{-j\beta r}}{4\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right] \rightarrow (14)$$

* Evaluation of field components

next finding the magnetic field by using maxwell's eq

The ϕ component of $\bar{H}$ is given by $\rightarrow (15)$ (1)

$$H_\phi = \frac{1}{\mu}(\nabla\times\bar{A})_\phi$$

But $(\nabla\times\bar{A})_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right].$

Now current elements placed along the z-axis

$$A_\theta = -A_z \sin\theta$$

$$A_r = 0$$

$$(\nabla \times \bar{A})_\phi = \frac{1}{r}\left[\frac{\partial}{\partial r}(r)\,(-A_z \sin\theta)\right] \rightarrow (16) \quad (2)$$

Substituting value of $(\nabla \times \bar{A})_\phi$ in eq 15, we get $\rightarrow (1)$

$$H_\phi = \frac{1}{\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}(-rA_z \sin\theta)\right] \rightarrow (17) \quad (3)$$

Substituting value of A_z from eq (14) in eq 17, we get

$$H_\phi = \frac{1}{\mu}\left[\frac{1}{r}\frac{\partial}{\partial r}\left\{-r\frac{\mu I_m e^{-j\beta r}}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin^2\theta}\right]\sin\theta\right\}\right]$$

$$= -\frac{I_m}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]\frac{d}{dr}\left[e^{-j\beta r}\right]$$

$$= \frac{-I_m}{2\pi\beta r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]\left[(e^{-j\beta r})(-j\beta)\right]$$

$$\therefore H_\phi = \frac{jI_m e^{-j\beta r}}{2\pi r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right] \quad (4) \quad (18)$$

The magnitude of the magnetic field strength for the radiation field of a halfwave dipole or quarter wave monopole is given by

$$|H_\phi| = \frac{I_m}{2\pi r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right] \quad (19) \cdot (5)$$

The electric field strength is related to magnetic field strength by the relation

$$\frac{E_\theta}{H_\phi} = \eta$$

For free space, $\eta = \eta_0 = 120\pi$

$\therefore E_\theta = (120\pi) H_\phi$

Substituting the value of H_ϕ from eq 18 we get

$$E_\theta = (120\pi) \left\{ \frac{j I_m e^{-j\beta r}}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] \right\}$$

$$\therefore E_\theta = \frac{j60 I_m e^{-j\beta r}}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] \longrightarrow 20.$$

The magnitude of the electric field strength for the radiation field of a halfwave dipole or a quarter wave monopole is given by

$$|E_\theta| = \frac{60 I_m}{r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right].$$

Radiation Resistance of a Short Dipole.

→ We shall calculate the radiation resistance of the short dipole shown in fig.

Radiation resistance is the fictitious resistance which when substituted in series with the antenna will consume the same power as actually radiated.

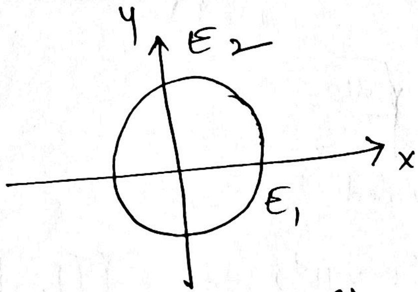


fig: Circularly polarized waves.

If surface integral of the average poynting vector is taken over any surface enclosing an antenna, the total power radiated by antenna is

$W = \int P_{av} \cdot ds$

W = power radiated in Watts

P_{av} = Average poynting vector in W/m^2

As the far field eq of antenna are easier than the near field relations, it is better to make the radius of the sphere large compared with the dimensions of the antenna.

Now power radiated by an antenna equal is equal to the avg power delivered to the antenna terminals which is nothing but $\frac{1}{2} I_m^2 R$ where I_m is the max current at the terminals and R is the radiation resistance at the terminals.

$$W = \frac{1}{2} I_m^2 R \quad \text{or} \quad R = \frac{2W}{I_m^2} \ \Omega.$$

The avg Poynting vector is $P_{avg} = \frac{1}{2} Re\,(E \times H^*)$

E_θ and H_ϕ are the radial component of the Poynting Vector.

$$P_r = \frac{1}{2} Re\,(E_\theta H_\phi^*)$$

H_ϕ^* is the complex conjugate of the H_ϕ^*

$$\frac{E_\theta}{H_\phi} = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 120\pi$$

$$\therefore P_r = \frac{1}{2} Re\,(\eta_0 H_\phi \cdot H_\phi^*) = \frac{1}{2} |H_\phi|^2 Re\,\eta_0$$

$\because Re\,\eta_0 = \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$$P_r = \frac{1}{2} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Hence the total Power radiated is

$$W = \int P_{avg} \cdot dg = \int \frac{1}{2} Re\,(E \times H^*) \cdot dg = \iint P_r \cdot dg$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{1}{2} Re\, E_\theta \cdot H_\phi^* \cdot dg = \frac{1}{2} \int_0^{2\pi}\int_0^{\pi} |H_\phi|^2 \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot r^2 \sin\theta \cdot d\theta \cdot d\phi.$$

$$= \frac{1}{2} \int_0^{2\pi} H_\phi \, d\phi \sqrt{\frac{\mu_0}{\varepsilon_0}} \int_0^{\pi} r^2 \sin\theta \cdot d\theta = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \int_0^{2\pi} d\phi \int_0^{\pi} \left(\frac{\omega \cdot I_m l \sin\theta}{4\pi c r}\right)^2 r^2 \sin\theta \cdot d\theta.$$

$$= \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} [2\pi] \frac{\omega^2 I_m^2 l^2 \sin^2\theta}{16\pi^2 c^2 r^2} \cdot r^2 \sin\theta \cdot d\theta.$$

$$= \frac{1}{16} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\omega^2}{c^2} \frac{I_m^2 l^2}{\pi} \int_0^{\pi} \sin^3\theta \cdot d\theta$$

$$= \frac{1}{16} \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\beta^2 I_m^2 l^2}{\pi} \cdot 2 \int_0^{\pi/2} \sin^3\theta \cdot d\theta.$$

$$= \frac{1}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\beta^2 I_m^2 l^2}{\pi}\cdot\frac{3-1}{3} = \frac{1}{8}\cdot\frac{2}{3\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\beta^2 I_m^2 l^2$$ (10)

$$W = \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{(\beta I_m l)^2}{12\pi} \text{ watt}$$

$$\because \int_0^{\pi/2} \sin^3\theta \cdot d\theta = \frac{3-1}{3} = \frac{2}{3}$$

Since $ds = r^2 \sin\theta \cdot d\theta \cdot d\phi$ and $\beta = \frac{\omega}{c}$

$$\therefore R = \frac{2W}{I_m^2}$$

Hence $$R_r = \frac{2\cdot\sqrt{\frac{\mu_0}{\varepsilon_0}}(\beta I_m l)^2}{I_m^2\, 12\pi}$$

$$= \sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{\beta^2 l^2}{6\pi} \qquad \because \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 120\pi$$

$$= 20\beta^2 l^2 = 20\left(\frac{2\pi}{\lambda}\right)^2 l^2 \qquad \because \beta = \frac{2\pi}{\lambda}$$

$$R_r = 80\pi^2\left(\frac{l}{\lambda}\right)^2 \Omega$$

$$= 789.57204\left(\frac{l}{\lambda}\right)^2$$

$$\simeq 790\left(\frac{l}{\lambda}\right)^2 \Omega.$$

If the complex poynting vector $= \frac{1}{2} E \times H^*$

$l = \lambda/2$

$$\therefore R_r = 790 \times\left(\frac{\lambda}{2}\times\frac{1}{\lambda}\right)^2$$

$$= 197.5\ \Omega$$

$$\simeq 197\ \Omega$$

Radiation resistance of Half wave Dipole.

The elemental area of the sphere shell is

$$ds = 2\pi r^2 \sin\theta \cdot d\theta$$

Total power $$w = \oint_s P_{avg} \cdot ds = \int_0^{\pi} \frac{30 I_{rms}^2}{\pi r^2}\left\{\frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}\right\}\cdot 2\pi r^2 \sin\theta \cdot d\theta.$$

$$= 60\, I_{rms}^2 \int_0^{\pi} \left\{ \frac{\cos^2 (\pi/2 \cos\theta)}{\sin\theta} \right\} . d\theta .$$

$$= 60\, I_{rms}^2 \int_0^{\pi} \frac{1}{2} \left\{ \frac{4 \cos (\pi \cos\theta)}{\sin\theta} \right\} d\theta . \quad \because 2\cos^2\theta = 4 \cos 2\theta .$$

$$= 60\, I_{rms}^2 . I$$

$$= \frac{1}{2} \int_0^{\pi} \left\{ \frac{4 \cos (\pi \cos\theta)}{\sin\theta} \right\} d\theta .$$

If $I = 1.219$

$$W = 60\, I_{rm}^2 \times 1.219$$

$$= 73.140\, I_{rm}^2$$

$$W_r = I_{rms}^2 \cdot R_r$$

$$R_r = 73.14 \simeq 73\, \Omega .$$

Effective length or Radiation Height (l_e) of linear Antenna

→ When the amplitude of the oscillator current in current element is assumed, then the expression for radiative field is given by equation

$$H_\phi = \frac{-I_m dl \sin\theta}{2\pi\lambda r} \sin\omega \left(t - \frac{r}{c}\right)$$

$$|H_\phi| = \frac{I_m d l}{2\pi r}$$

The actual length of shorter antenna may be assessed by finding the mean value of current over the length assumed and accordingly expression reduces to

$$|H_\phi| = \frac{I_{av} l}{2\pi r}$$

Where I_{av} = Average current

I = length of short or linear antenna, as against current element length of dl.

Since equivalent or radiation length l_e will be $(l/2)$ as the mean value of current is $(I_m/2)$ i.e by putting

$$I_{av} = \frac{I_m}{2}$$

$$|H_\phi| = \frac{l}{2\lambda r} \cdot \frac{I_m}{2} = \frac{I_m}{2\lambda r} \cdot \frac{l}{2}$$

$$|H_\phi| = \frac{I_m l_e}{2\lambda r}$$

where $l_e = l/2$

= Effective length or height of the antenna.

Sinusoidal current or Co-sinusoidal distribution.

The current distribution is

$$I = I_m \sin\left(\frac{x}{l/2}\right)\frac{\pi}{2} = I_m \sin\left(\frac{\pi x}{l}\right)$$

This gives instantaneous current at point x from the end.

x lies between 0 to $l/2$.

$$I_{av} = \frac{\int_0^{\pi/2} I_m \sin\theta \, d\theta}{\int_0^{\pi/2} d\theta} = \frac{I_m[-\cos\theta]_0^{\pi/2}}{[\theta]_0^{\pi/2}} = \frac{I_m(+1)}{\pi/2} = \frac{2I_m}{\pi}$$

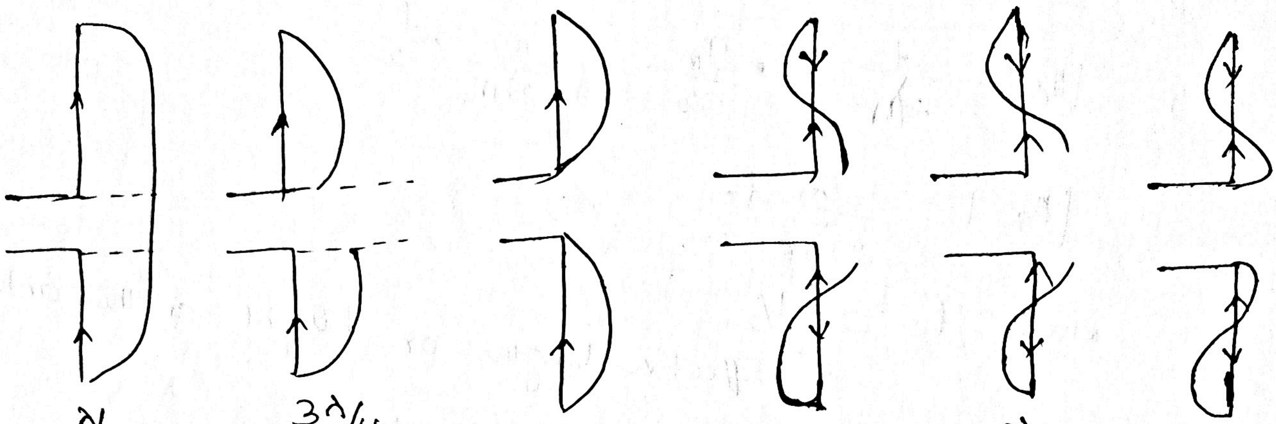
$$I_{av} = \frac{2I_m}{\pi}$$

Thus for sinusoidal or cosinusoidal current distribution effective height is obtained from

$$|H_\phi| = \frac{l}{2\lambda r} \cdot I_{av} = \frac{l}{2\lambda r} \cdot \frac{2I_m}{\pi} = \frac{I_m}{2\lambda r} \cdot \frac{2l}{\pi}$$

$$\boxed{|H_\phi| = \frac{I_m}{2\lambda r} \cdot l_e \quad \text{where } l_e = \frac{2l}{\pi}}$$

Natural Current Distributions of Thin Linear Wire Antennas of Various Lengths:



$\lambda/2$ $3\lambda/4$ λ $5\lambda/4$ $3\lambda/2$ 2λ

fig: Natural current distributions in thin linear antennas of different length

→ conductor diameter is less than $\lambda/140$, such antenna is called thin wire antenna.

Applications of Network Theorems to Antenna

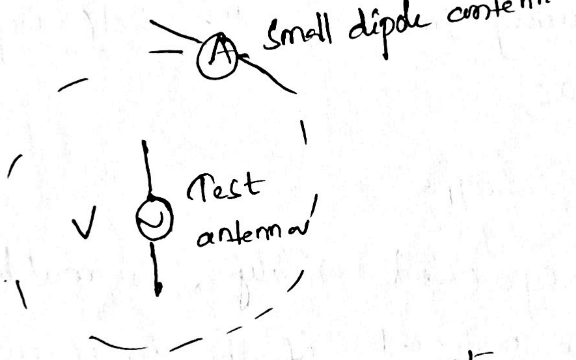
→ The properties of the Tx antenna and the Rx antenna are related to each other through various antenna theorems.

→ Current approximation is not sinusoidal in Rx antenna. and impedance property, directional property are not same for the Tx and Rx conditions.

Equity of Directional patterns

Statement: The directional pattern of an antenna as a receiving antenna is identical to that when used as a transmitting antenna.

Proof: The above mentioned antenna theorem is the outcome of the application of the reciprocity theorem used in the linear and bilateral networks. Basically a directional pattern of a transmitting antenna is represented as a polar characteristic because it indicates the strength of the radiated field at a fixed distance in several directions in the space. Similar to this directional pattern of a receiving antenna is also a polar characteristic which indicates the response of the antenna for unit field strength from different directions.



Directional pattern measurement for a transmitting antenna.

To measure the directional pattern of an antenna as a transmitting antenna, the test antenna is kept at the centre of very large sphere and the small dipole antenna is moved along the surface of this sphere shown in fig.

A voltage V is connected to the test antenna, placed at the centre of imaginary sphere and the current I flowing in short dipole antenna is measured using ammeter at different positions. This current is the measure of the electric field at different positions of the dipole antenna. Now using the reciprocity theorem, the positions of the voltage excitation and the current measurements are interchanged. Now the same voltage V is applied to the terminals of the small dipole antenna, which is moved along the surface of the sphere and the current I is measured in the test antenna located at the centre. Then the receiving pattern for the test antenna can be obtained. But according to the reciprocity theorem, for every location of the dipole antenna, the ratio of V to I is same as before obtained for the test antenna as a transmitting antenna. Thus the radiation pattern i.e directional pattern of a receiving antenna is identical to that of the transmitting antenna.

Equivalence of Transmitting and Receiving Antenna Impedances

Statement: The impedance of an isolated antenna used for transmitting antenna as well as receiving purposes is identical.

Proof: Consider two antenna terminals namely A_1 and A_2. Antenna A_2 is located far away from A_1, the self impedance of antenna A_1 can be written as,

$$A_1 = Z_{S_1} = \frac{V_1}{I_1} = Z_{11}.$$

Now, when two antennas are separated widely, the mutual impedance Z_{12} of the antenna A_1 can be neglected if the A_1 is used as transmitting antenna. But if A_1 is used as receiving antenna, the mutual impedance Z_{12} can not be neglected, as it is the only parameter indicating coupling between two antennas. So the load Z_L is connected to the antenna A_1 used as receiving antenna. Similarly coupling between A_1 and A_2 is represented with the help of mutual voltage $Z_{12}I_2$, which is due to the mutual impedance Z_{12} and the current I_2 in antenna A_2.

$$V_{OC} = Z_{12} I_2$$

Under short ckt condition, current flowing from terminal 1 to 1' is given by

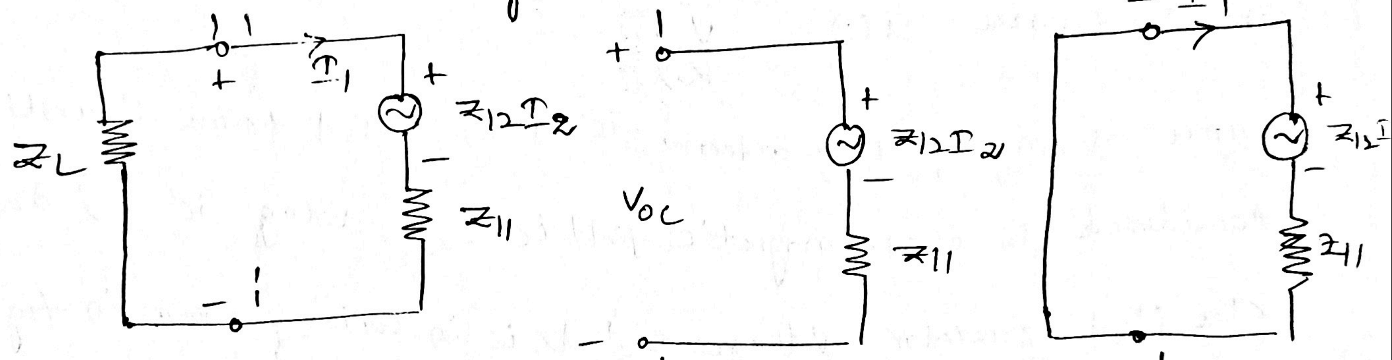
$$I_{sc} = \frac{Z_{12} I_2}{Z_{11}}$$

Hence the ratio of V_{oc} to I_{sc} is called as transfer impedance.

$$\frac{V_{oc}}{I_{sc}} = \frac{Z_{12} I_2}{\left(\frac{Z_{12} I_2}{Z_{11}}\right)} = Z_{11}.$$

$Z_{12} I_2$ is the generator with internal impedance Z_{11}.

Hence from above equation the receiving antenna impedance is equal to the transmitting antenna impedance.



a) Loaded condition. b) open ckt condition c) short ckt condition.

Equality of Effective lengths: l_{eff}.

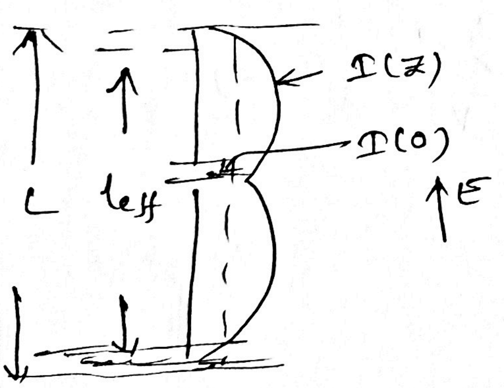


fig: Representation of Effective length

The effective length of the antenna is defined as the length of an equivalent linear antenna which has current $I(0)$ along its length at all points radiating the field strengths in direction ⊥ to the length same as the actual antenna. The current $I(0)$ is the current at the antenna terminals.

For the transmitting antenna

$$I(0)\, l_{eff}\,(\text{trans}) = \int_{-L/2}^{+L/2} I(z)\,dz$$

$$l_{eff}(\text{trans}) = \frac{1}{I(0)} \int_{-l/2}^{+l/2} I(z)\,dz.$$

For the receiving antenna the effective length is defined as the ratio of the open ckt voltage developed at the antenna terminals to the given received field strength. Hence

$$l_{eff}(\text{rec}) = \frac{-V_{oc}}{E}$$

Let Z_a be the antenna impedance, measured at the antenna terminals. Assume that the voltage V is applied at the antenna terminals shown. Then the current $I(0) = \frac{V}{Z_a}$

Current at any point of antenna is $I(z)$ called prime situation. considered the electromagnetic field is E_z', votage is $E_z'.dz$.

The ideal generator voltage $E_z'.dz$ is in series shown in fig.

The ideal generator produces a current dI_{sc} in the antenna, called double prime situation

According to the reciprocity theorem

$$\frac{V}{I(z)} = \frac{E_z'.dz}{dI_s}, \quad dI_{sc} = \frac{E_z'.dz}{V_z}.I(z).$$

$$I_{sc} = \frac{1}{V}\int E_z' I(z).dz.$$

according to Thevenin's theorm,

$$V_{oc} = -I_{sc}.Z_a$$

$$= -\frac{Z_a}{V}\int E_z' I(z).dz$$

$$= \frac{-1}{I(0)}\int E_z' I(z).dz.$$

$$V_{oc} = \frac{-E_z}{I(0)} \int I(z).dz.$$

Hence $$\frac{-V_{oc}}{E_z} = \frac{1}{I(0)} \int I(z).dz.$$

Hence from above equations it is clear that the effective length of an antenna used for receiving purpose is equal to the effective length of if it is used for the transmitting purpose.

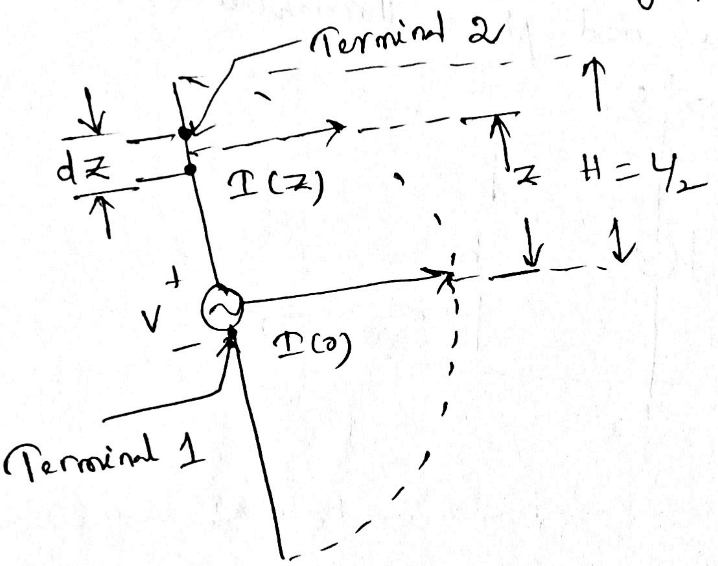


fig: Transmitting antenna

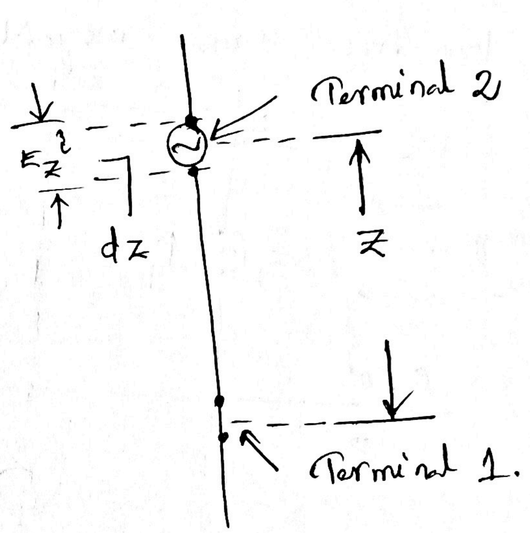
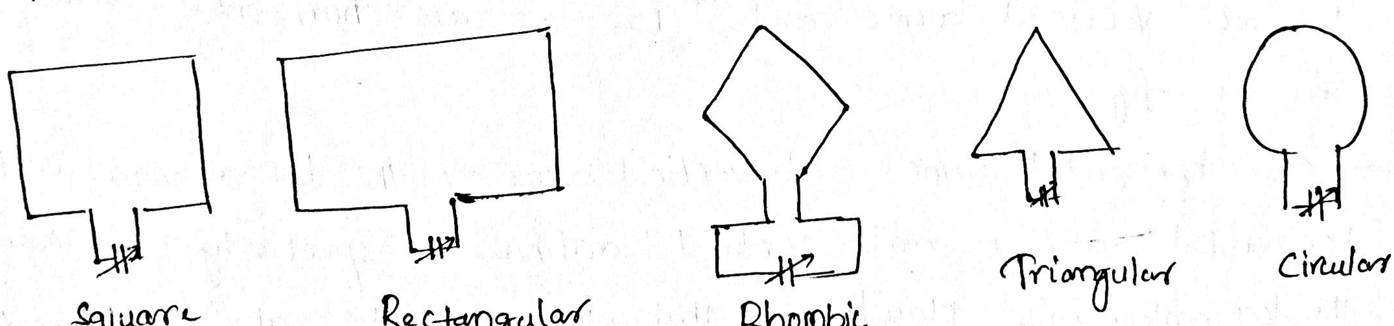


Fig: Receiving antenna.

* Loop Antenna:



Generally, a loop antenna nothing but a radiating coil of any shape with one or more turns carrying a r.f current. The loop antenna may assume any one of the shapes as shown in above fig.

Generally a loop may consist one turn or more turns on a ferrite or air core. If a loop consist more than one turn then it is commonly called as frame. Generally the loop antennas are of two types classified on the basis of dimensions of the loop. In the first type the dimensions of the loop are very small as compared with one wavelength. In other type, the dimensions of the loop are comparable with one wavelength. For such loop antennas the convinient assumption made is that the current in the loop has same magnitude and phase throughout.

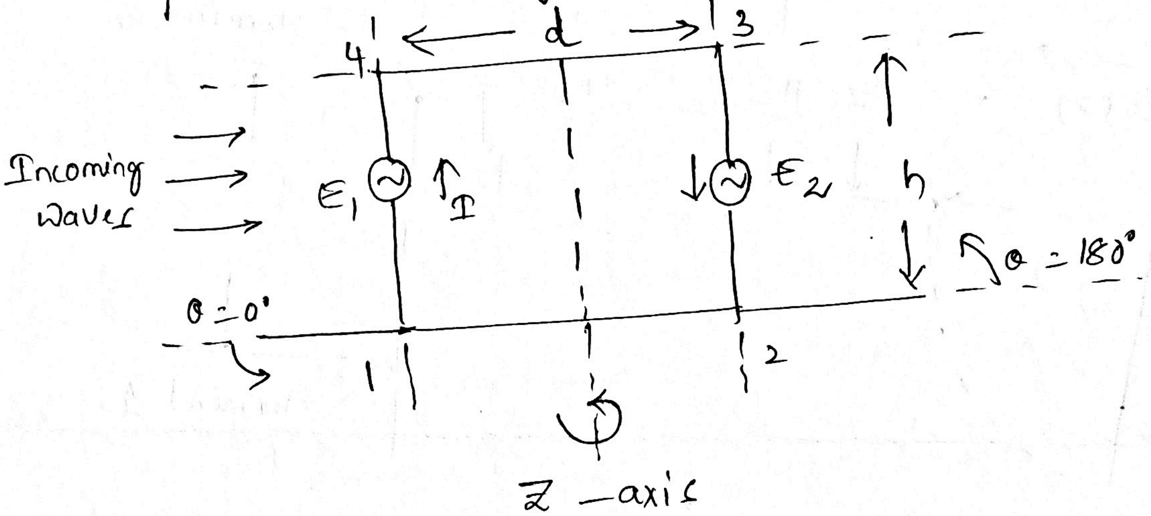


fig: Loop antenna

Consider a rectangular loop with the turn with sides 23 and 14 as vertical arms and 12 and 43 as horizontal arms shown in the fig.

→ The horizontal arms and vertical arms of the loop antenna acts as horizontal antenna and vertical antenna respectively. Consider the loop antenna is placed such that its plane is at right angles to the direction of wave travel, if the incoming waves are vertically polarised, then the voltage will be induced in two vertical arms of the loop. These voltages are same in magnitude but as the current in two vertical arm set due to two voltages is in opposite direction in two arms, the two voltages get cancelled out.

Directivity:- The radiation pattern of the dipole antenna is more directional as its length increases. When the overall length is greater than about one wave length, the number of lobes increases and the antenna loses directional properties. The parameter that is used as figure of merit.

Mathematically the directivity can be represented as:

$$D_0 = 4\pi . \frac{F(\theta, \phi)|_{max}}{\int_0^{2\pi}\int_0^{\pi} F(\theta, \phi) \sin\theta \cdot d\theta \cdot d\phi}$$

$F(\theta,\phi)$ is related to the radiation intensity U

$$U = B_0 \cdot F(\theta, \phi)$$

The length of dipole antenna L has

$$F(\theta, \phi) = F(\theta) = \left[\frac{\cos(\pi/2 \cos\theta) - \cos(\pi/2)}{\sin\theta} \right]^2$$

$$U_0 = \eta \frac{|I_0|^2}{8\pi^2}$$

$$D_0 = \frac{2\ F(\theta)|_{max}}{\int_0^{\pi} F(\theta) \cdot \sin\theta \cdot d\theta}$$

$$= \frac{2\ F(\theta)|_{max}}{Q}$$

Maximum effective area are related to directivity

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$