ANTENNAS AND WAVE PROPAGATION

R20

UNIT – I : Antenna Fundamentals

- Introduction to Antennas
- Radiation Mechanism – single wire, 2 wire, dipoles.
- Current distribution on thin wire antenna
- Characteristics of Antenna – Radiation Pattern, Radiation intensity, Beam Solid Angle, Directivity, Gain, Polarization, efficiency, Equivalent areas, Radiation resistance, Effective length, Antenna Temperature.
- Relation between Maximum Directivity and effective area
- Illustrated Problem.

ANTENNAS AND WAVE PROPAGATION

UNIT-I ANTENNA FUNDAMENTALS

* Introduction:

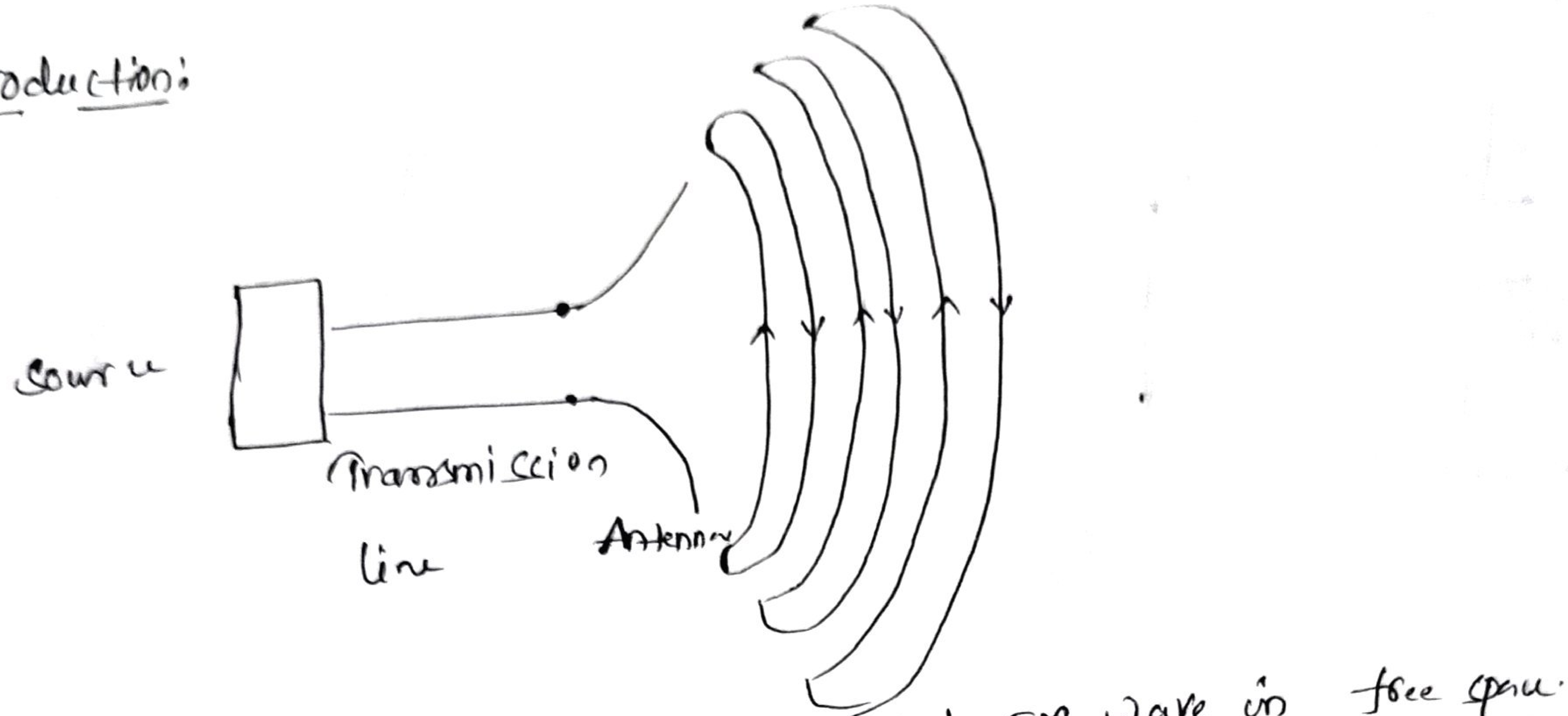


Fig : Antenna as a matching device between free space and wave launching system.

→ The electric charges are the sources of the electromagnetic (EM) fields. When these source are time varying, the EM waves propagate away from the source and the radiation takes place.

→ In general, the radiation can be considered as a process of transmitting energy. The radiation of the electromagnetic wave into the space is effectively achieved by using a conducting or dielectric structures called antennas or radiators.

Defenition of an Antenna:

(A) A metallic device used for radiating or receiving radio waves is called antenna.

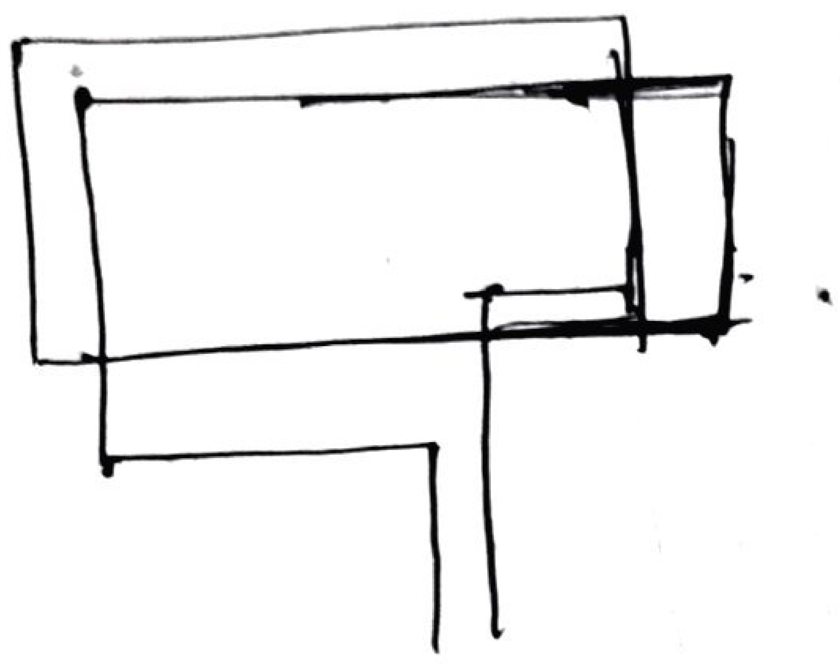
(B) According to IEEE standard, the defenition of an Antenna is radiating or receiving radio waves.

(C) Antenna is regarded as a transition between the free space and a system used for launching an electromagnetic waves.

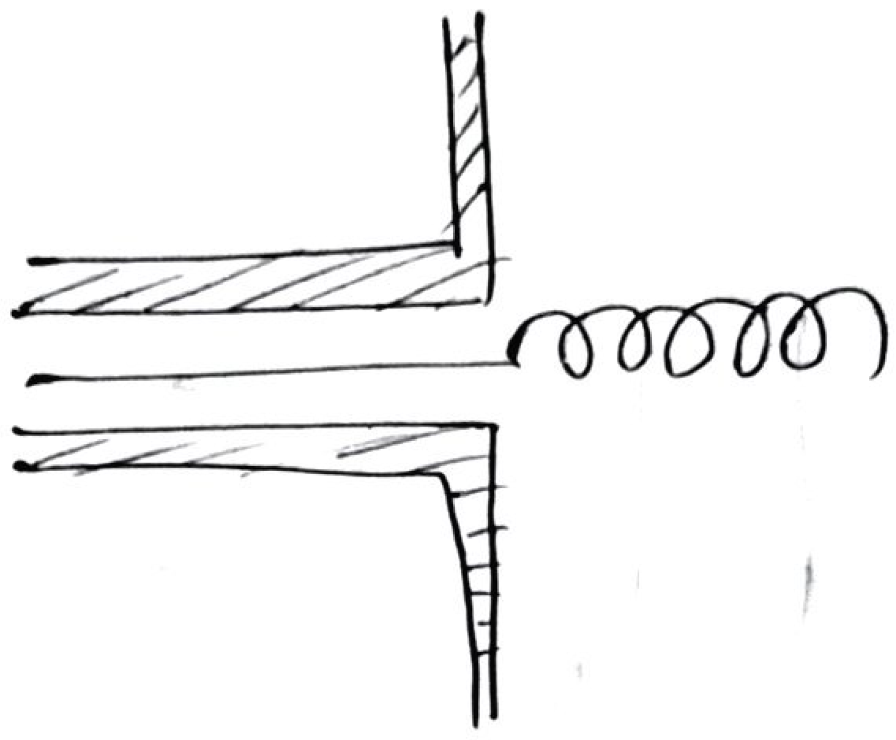
→ The system used for launching the electromagnetic waves either transmission line or wave guide.

Let us show the different types of antenna.

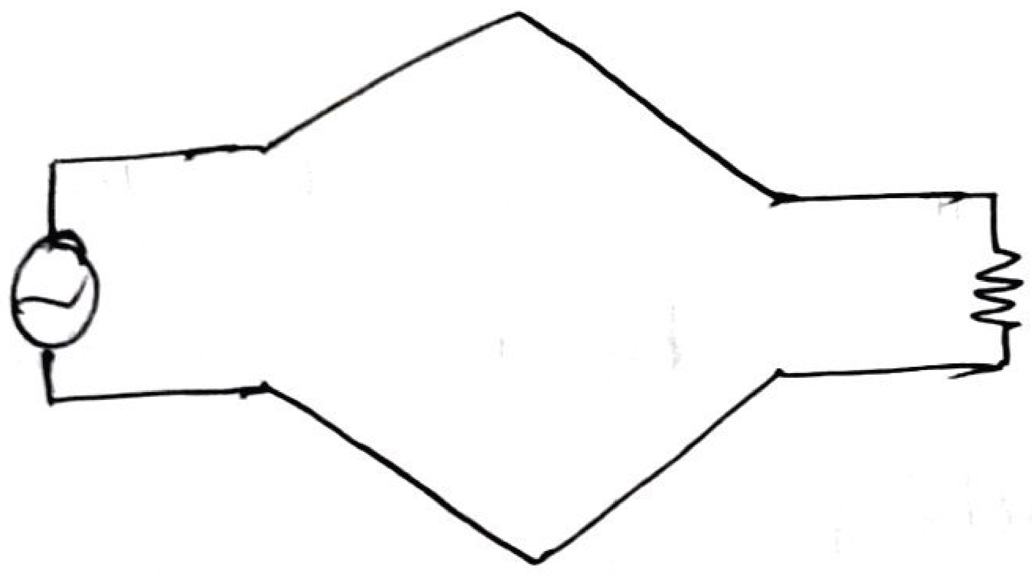
a) Dipole



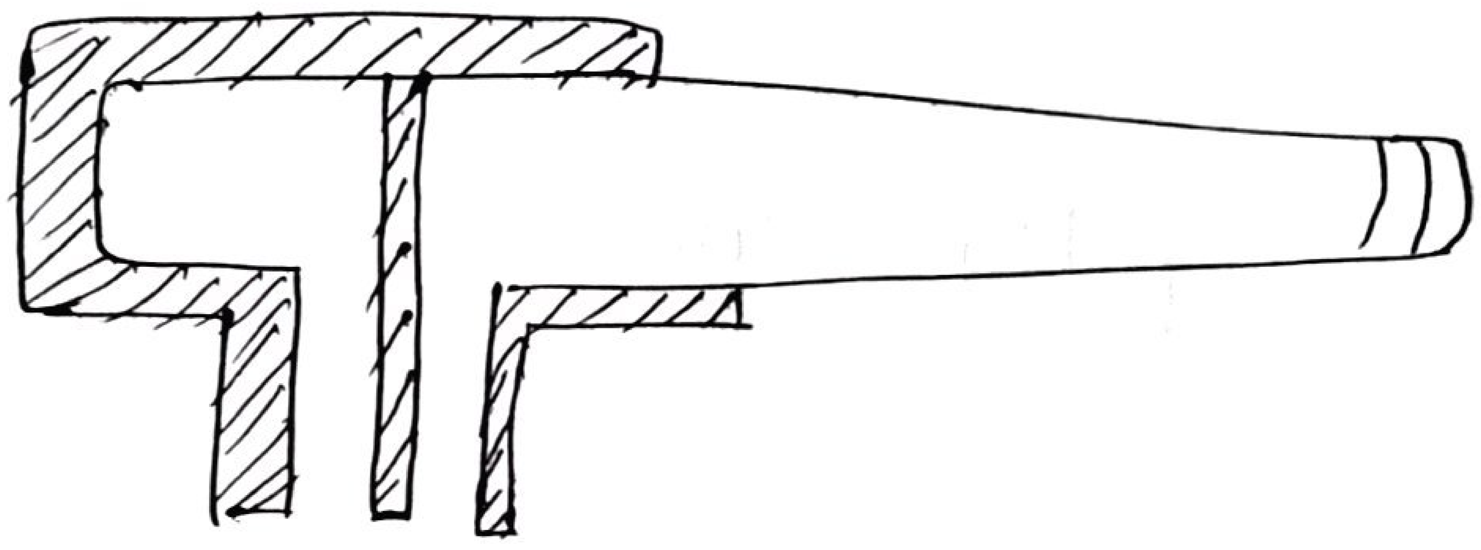
(b) loop



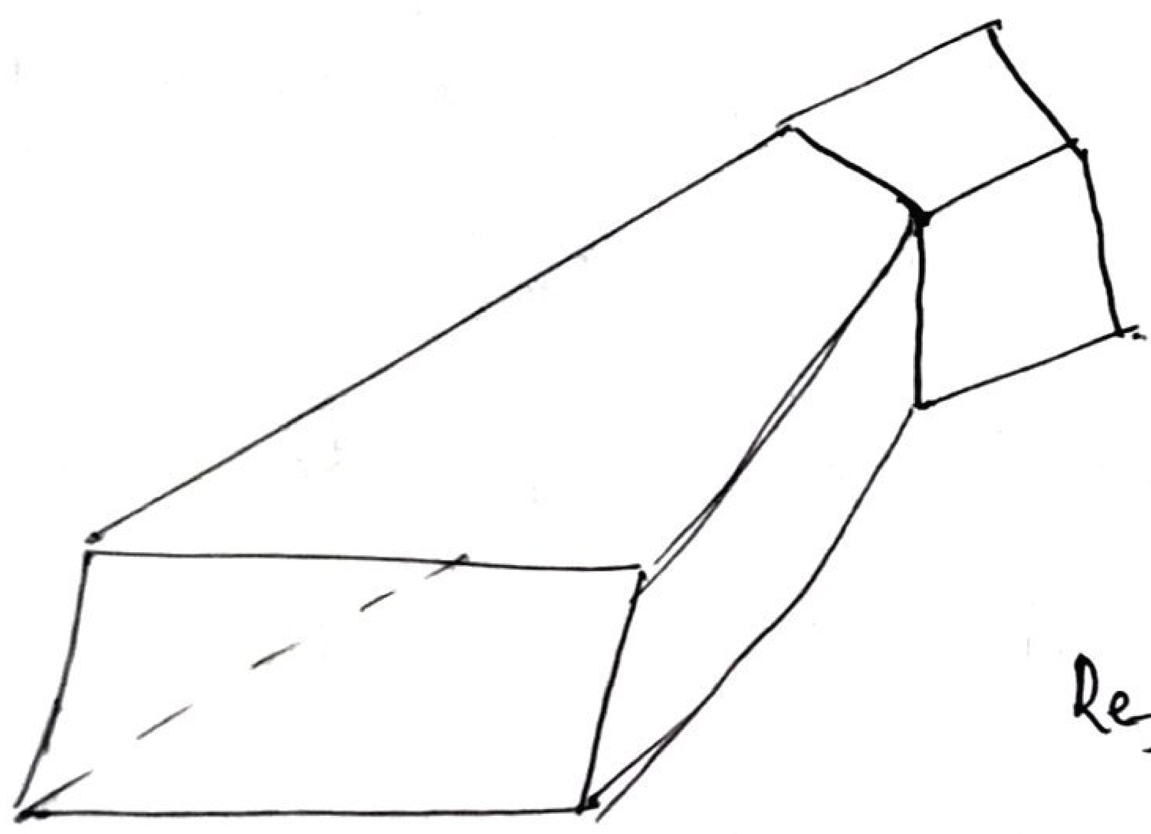
c) Helical



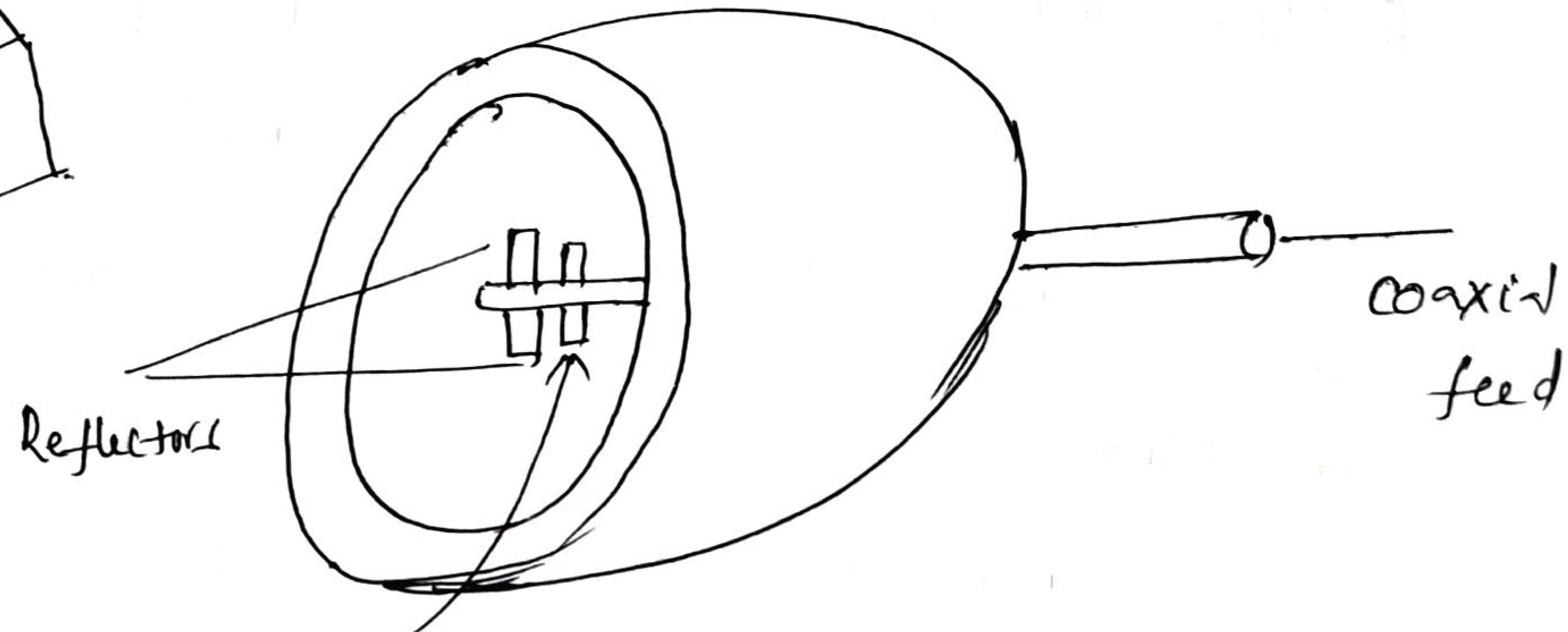
d) Rhombic



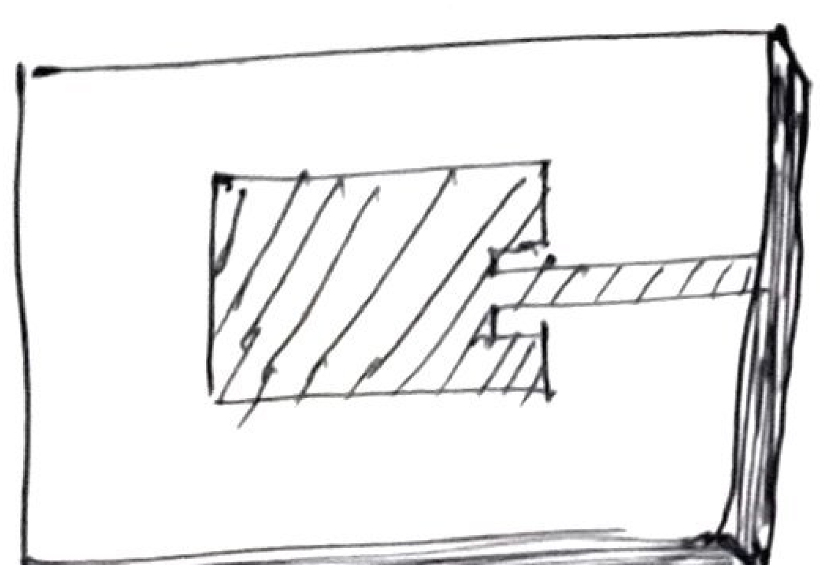
e) Dielectric



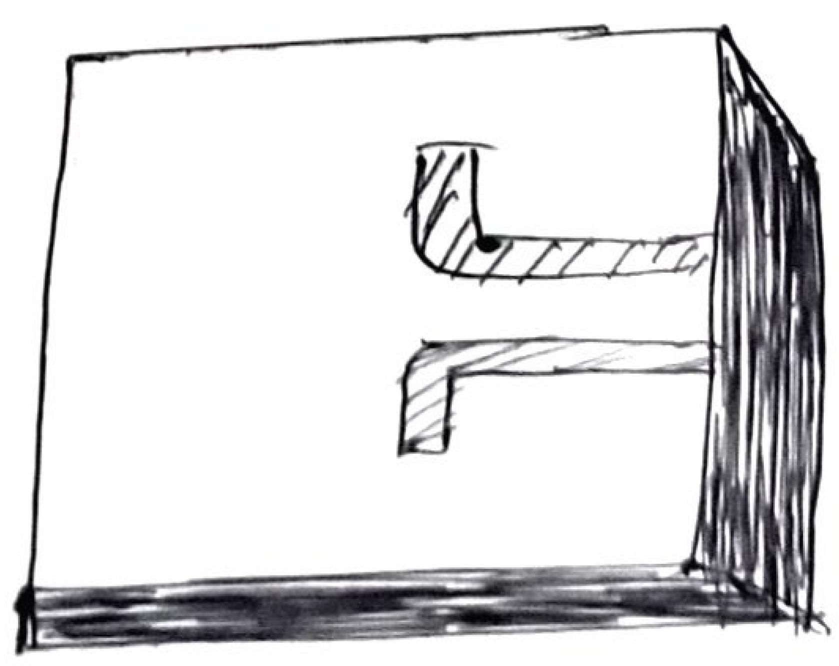
f) Pyramidal horn



g). Parabolic dish reflector



h) Microstrip Patch antenna



i) Co-planar strip.

→ The most commonly used antenna is the dipole antenna. It is made up of two straight wires or conductor laying along the same axis.

→ The loop loop antenna is consist a single turn or many turns of wire forming a loop. It is generally excited by a generator directly.

→ The antenna with a wire in the form a helix backed by a ground plane is called helical antenna.

→ Dipole antenna, loop antenna, helical antennas are wire antennas, which are used in aircrafts, ships, automobilies

→ In travelling wave antenna, the antenna is designed in one direction, in travelling wave.
The velocity of this wave equals the velocity of light, and excites the waves in the space in same direction strongly., so maximum directivity can be achieved.

Ex: Rhombic antenna.

→ Half of the rhombic antenna is called, V antenna, the travelling wave is guided by a dielectric is called dielectric antenna.

→ Example of aperture antenna is horn antenna. This is also called, electromagnetic horn.

→ Parabolic reflectors are commonly used for microwave radiation.

→ The antenna to be used with microwave integrated circuit may be placed on a dielectric substrate. As antenna is placed on a dielectric substrate, the integrated circuit type antenna is also called patch antenna.

Radiation Mechanism:

→ Different types of basic causes of radiation are

Single Wire:

Radiation must be time varying current in single wire antennas

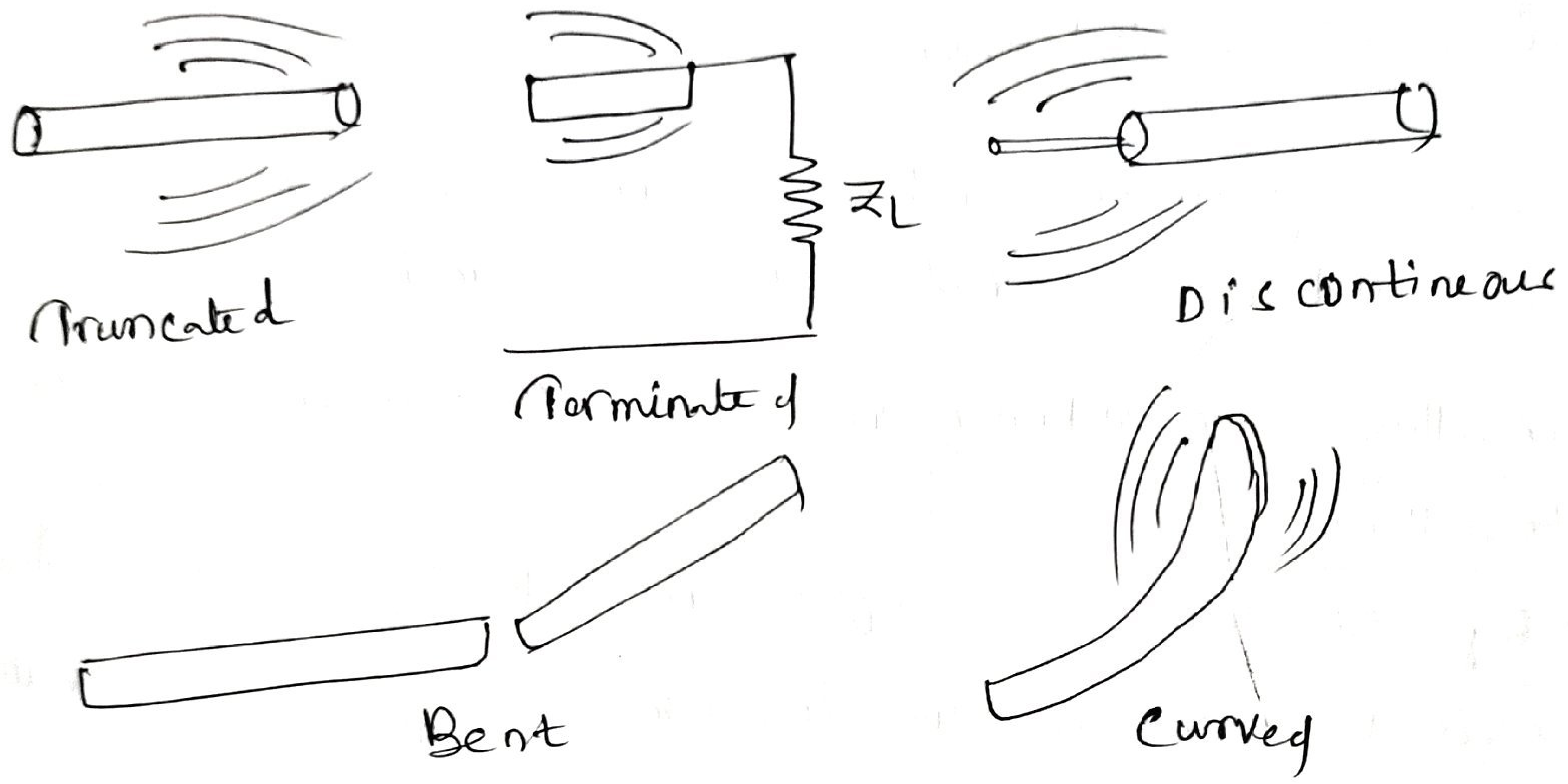


Fig: Radiation from Single wire.

→ If charge is stationary, no current is developed. So no ratiation is observed.

→ If charge is moving with uniform velocity then

i) No radiation for straight which is ∞ in extent.

ii) Possible radiation if wire is curved, discontineous, terminated or truncated.

Two wires:

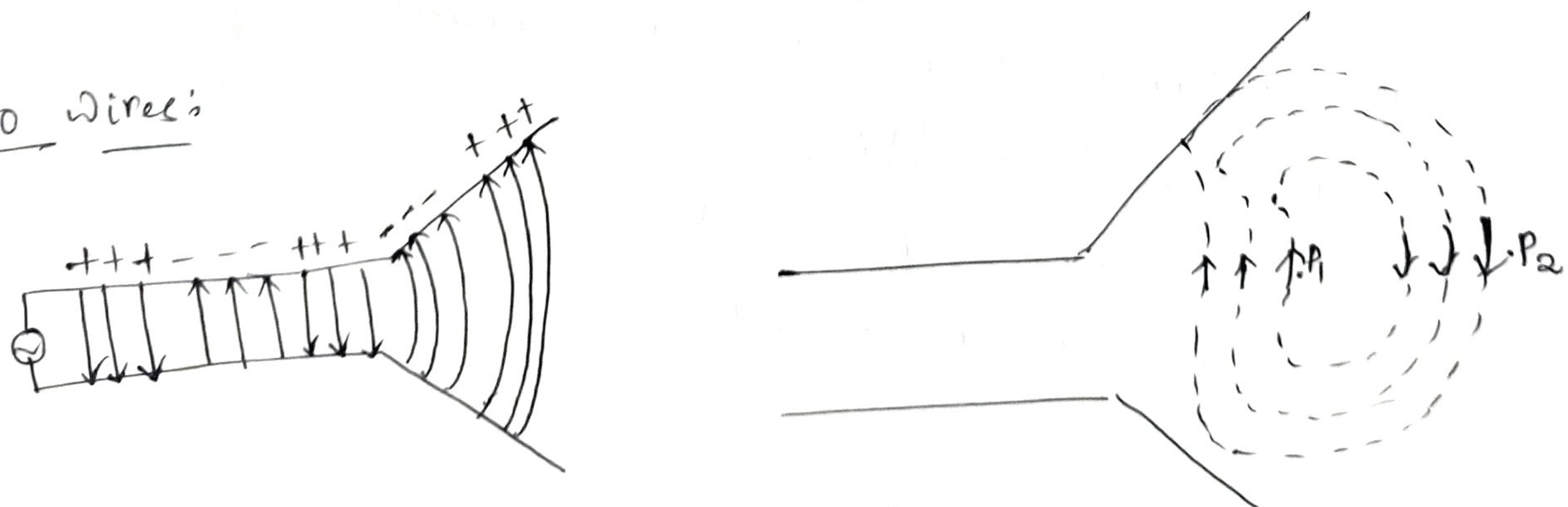


Fig: Radiation from two wire antennas.

→ When the source is applied, electric field gets developed between conductors. The strength of electric lines of force is proportional to the electric field intensity.

→ Due to charge movement current is produced and it produces magnetic field intensity.

→ Assuming sinusoidal Voltage between conductors, the electric field between the conductors is also sinusoidal.

<u>Dipole:</u>

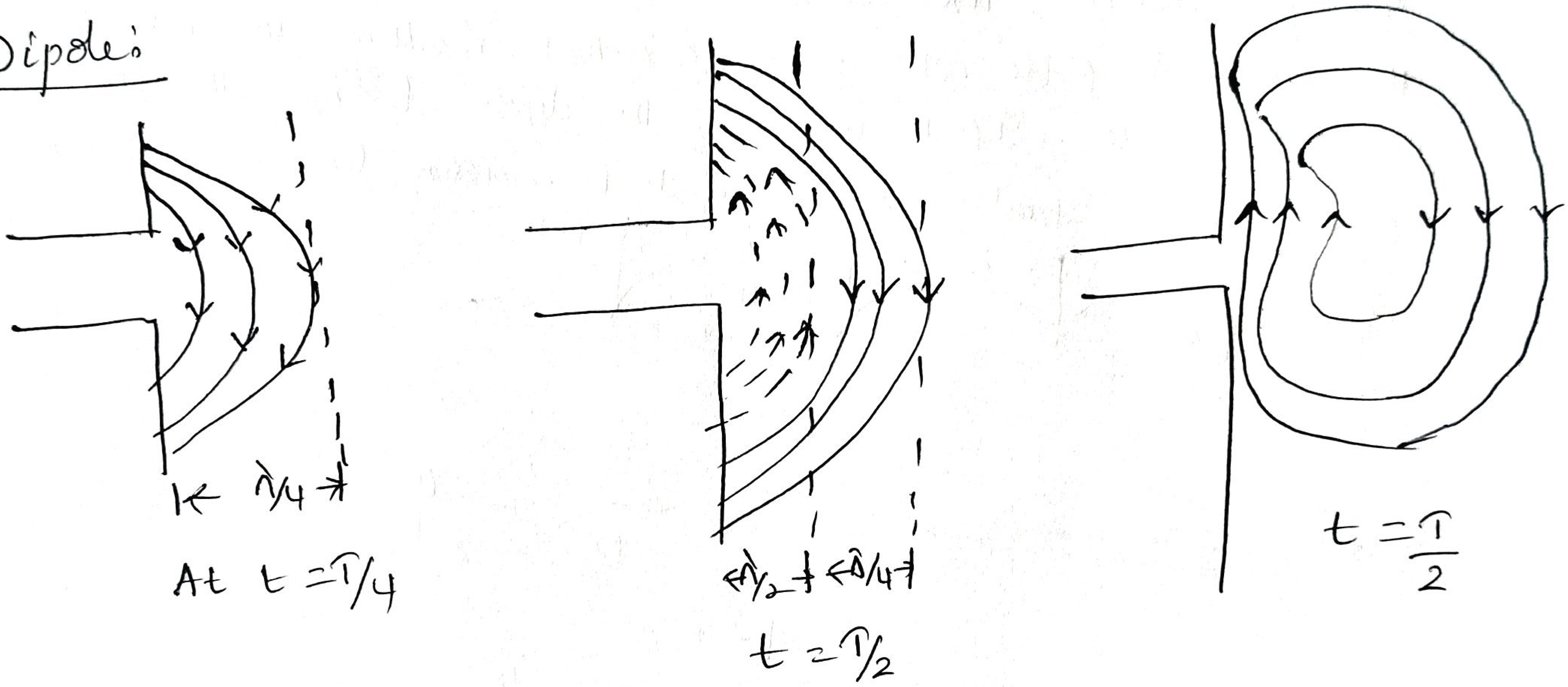


Fig: Formation of electric field line for short Dipole

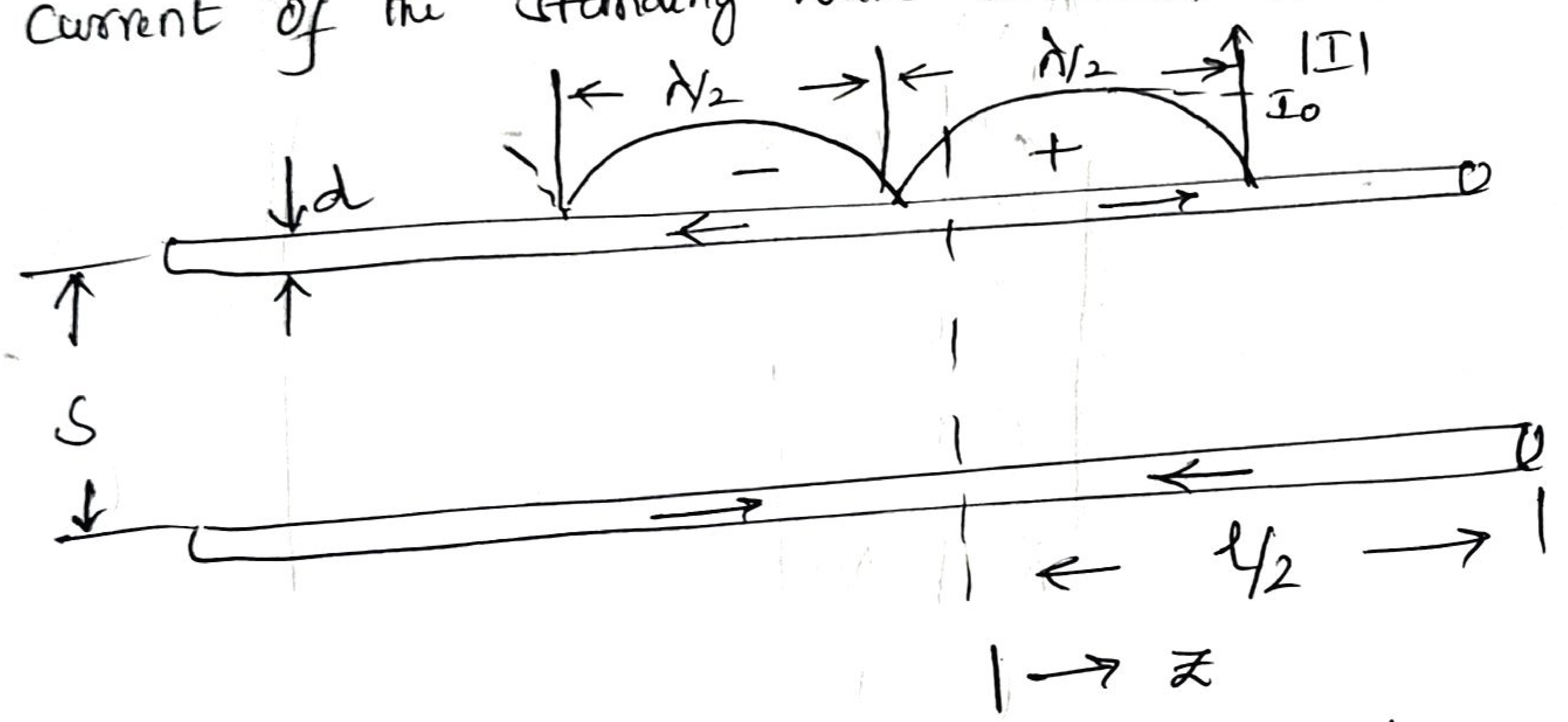
→ Small dipole is centre fed in the first quarter of the period T i.e $t = T/4$. Here charge attains maximum value. At this instant, the lines travel radially outward a distance $\lambda/4$.

→ Next travel additional distance of $\lambda/4$, the total distance is $\lambda/2$. The charge density on the opposite conductors starts diminishing. This is because opposite charges are starting. The opposite charges neutralize the charges on the conductor. Which is also produce the three lines of force which travel a distance $\lambda/4$ during second quarter of the first half.

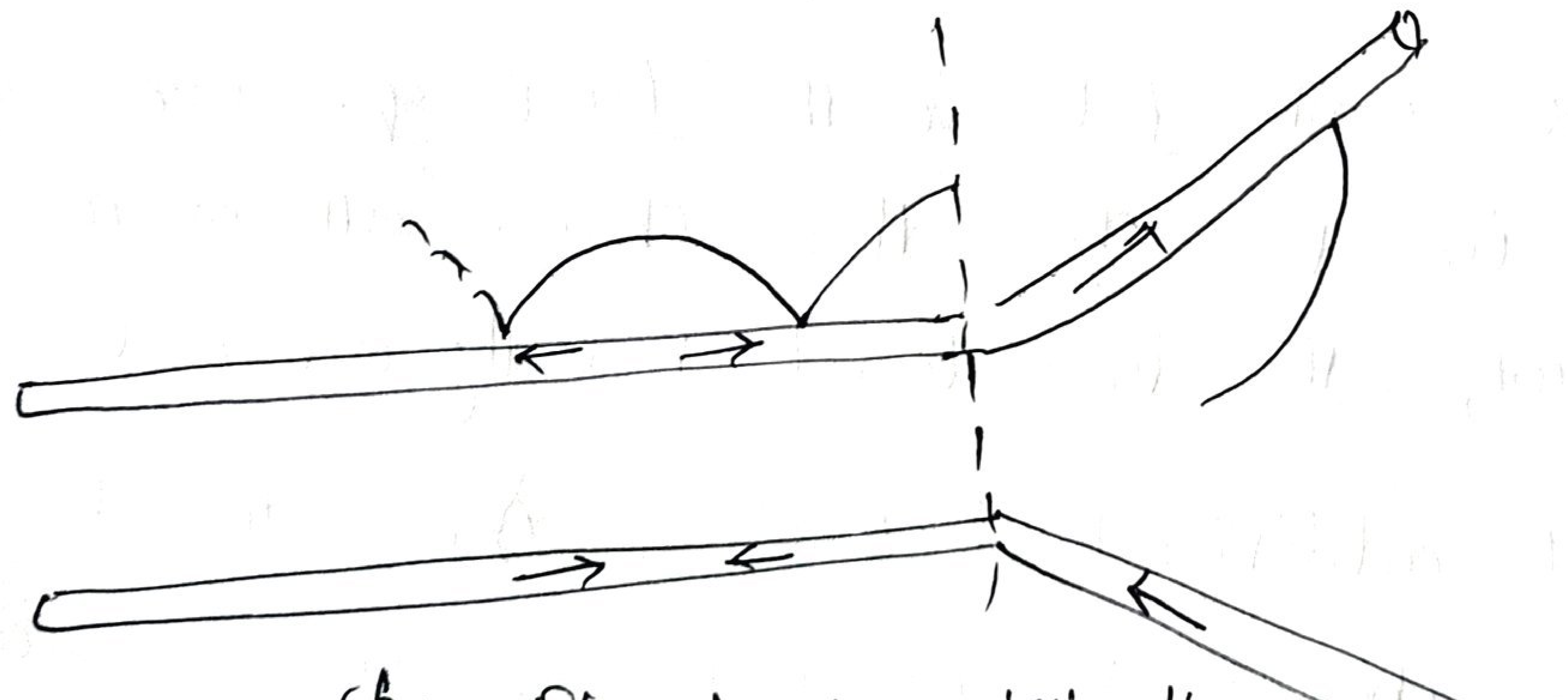
Current Distribution on thin wire Antenna: —

Let us consider a Lossless two wire transmission line in which the movement of charges creates a current having a value I with each wire. This current at the end of the transmission line is reflected back, when the transmission line has parallel end points resulting in formation of standing waves in combination with incident wave.

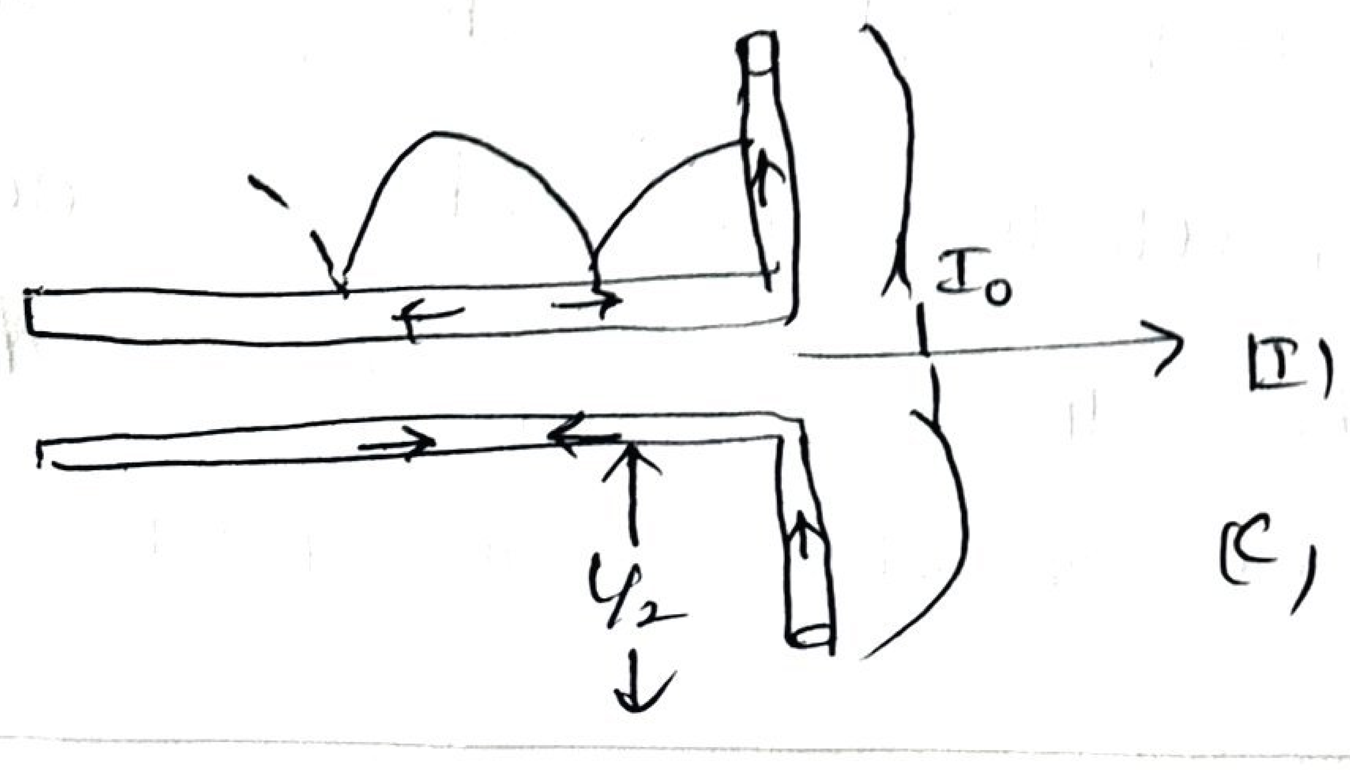
When the transmission line is flared out at 90° forming geometry of dipole antenna, the current distribution remains unaltered and the radiated fields not getting cancelled resulting in net radiation from the dipole. If the length of the dipole $L < \lambda/2$, the phase of current of the standing wave in each transmission line remains same.



(a) Two wire transmission line



(b) flared transmission line



(c) Linear Dipole

fig: Current distribution on Lossless two-wire transmission line, flared transmission line and linear dipole.

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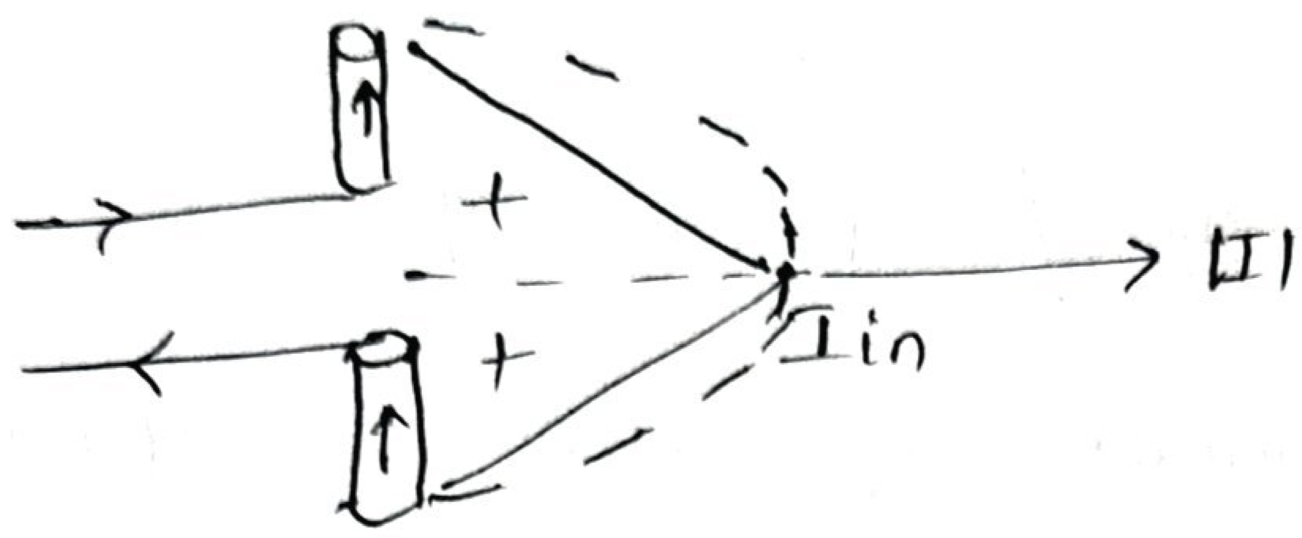
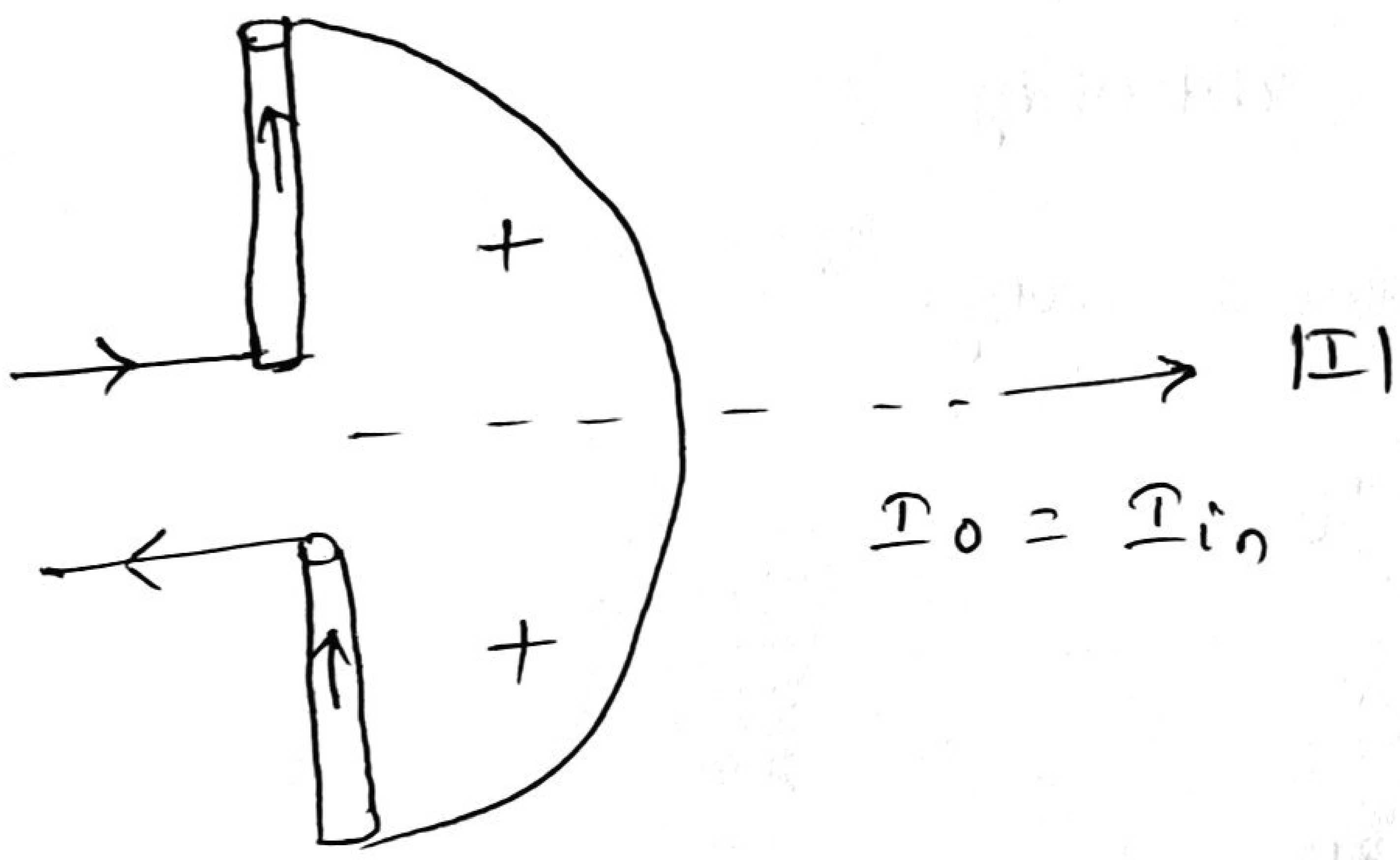
If diameter of each line is small $d < \lambda/2$, the current distribution along the line will be sinusoidal with null at end but overall distribution depends on the length of the dipole

The current distribution for dipole of the length $l << \lambda$



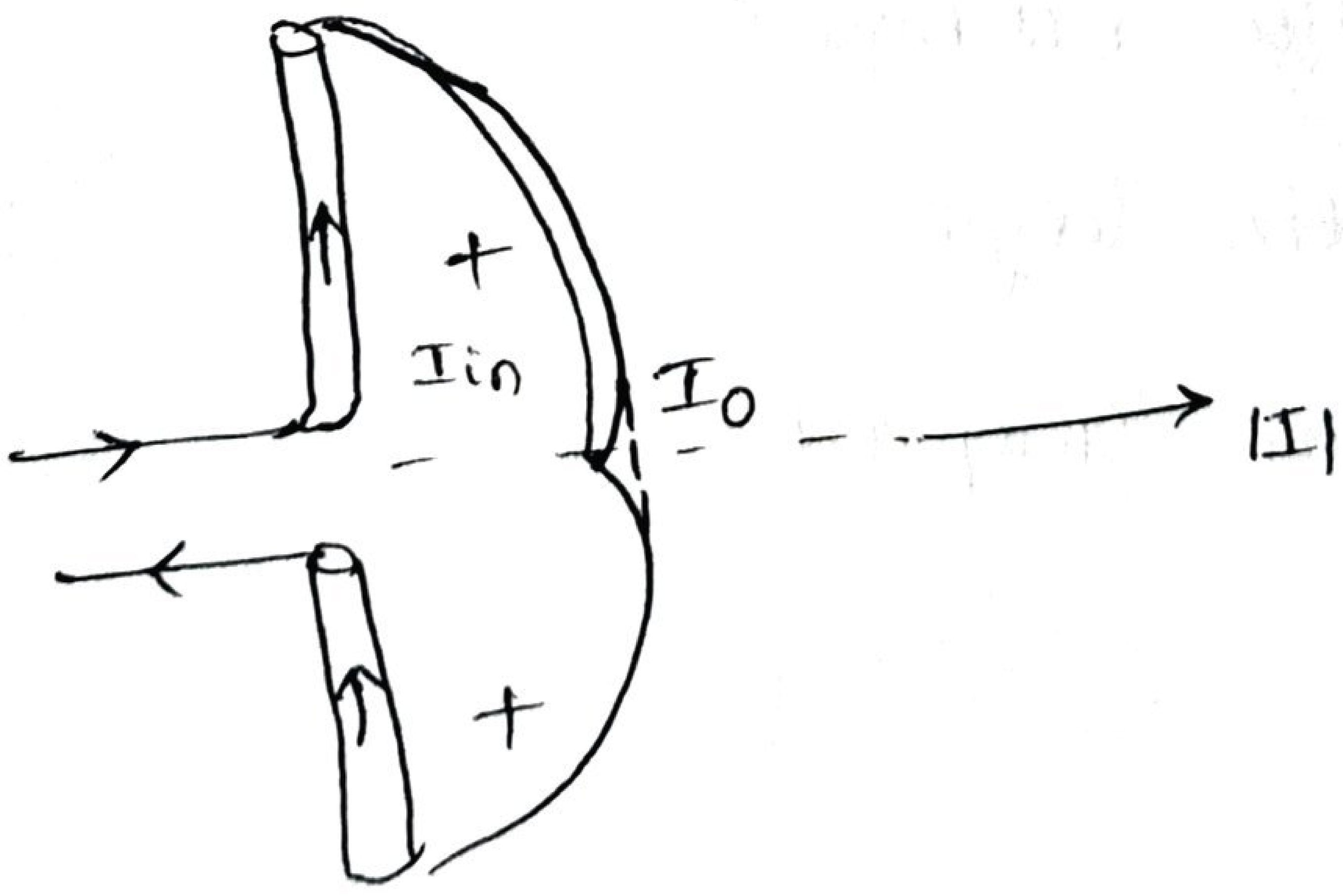
(a) $l << \lambda$

for $l = \lambda/2$



(b) $l = \lambda/2$

for $\lambda/2 < l < \lambda$



(c) $\lambda/2 < l < \lambda$

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When $l > \lambda$, the current goes phase reversal between adjoining half-cycles. Hence, current is not have same phase along all parts of transmission line. This will result into reference and canceling effects in the total radiation pattern.

Charactoristics of antenna / Antenna Parametors :-

To describe the performance of antenna, defenitions of various parametors are necessary. They are

* Radiation Pattern
* Radiation intensity
* Beam solid angle
* Directivity
* Gain
* Polarization
* Effeciency
* Equivalent areas
* Radiation resistance
* Effective length
* Antenna Temparature

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Radiation Pattern

→ An antenna radiation pattern or antenna pattern is defined as a mathematical function or graphical representation of radiation properties of an antenna as a function of space coordinates.

→ In most cases radiation pattern is determined from the far field regions and it represented as a function of directional coordinates.

→ It includes power density, radiation intensity, field strength, directivity, polarization.

→ It is most two or three dimensional.

→ A trace of the received power at constant radius is called power pattern.

→ Graph of the spatial variation of the electric or magnetic field along a constant radius is called an amplitude field pattern.

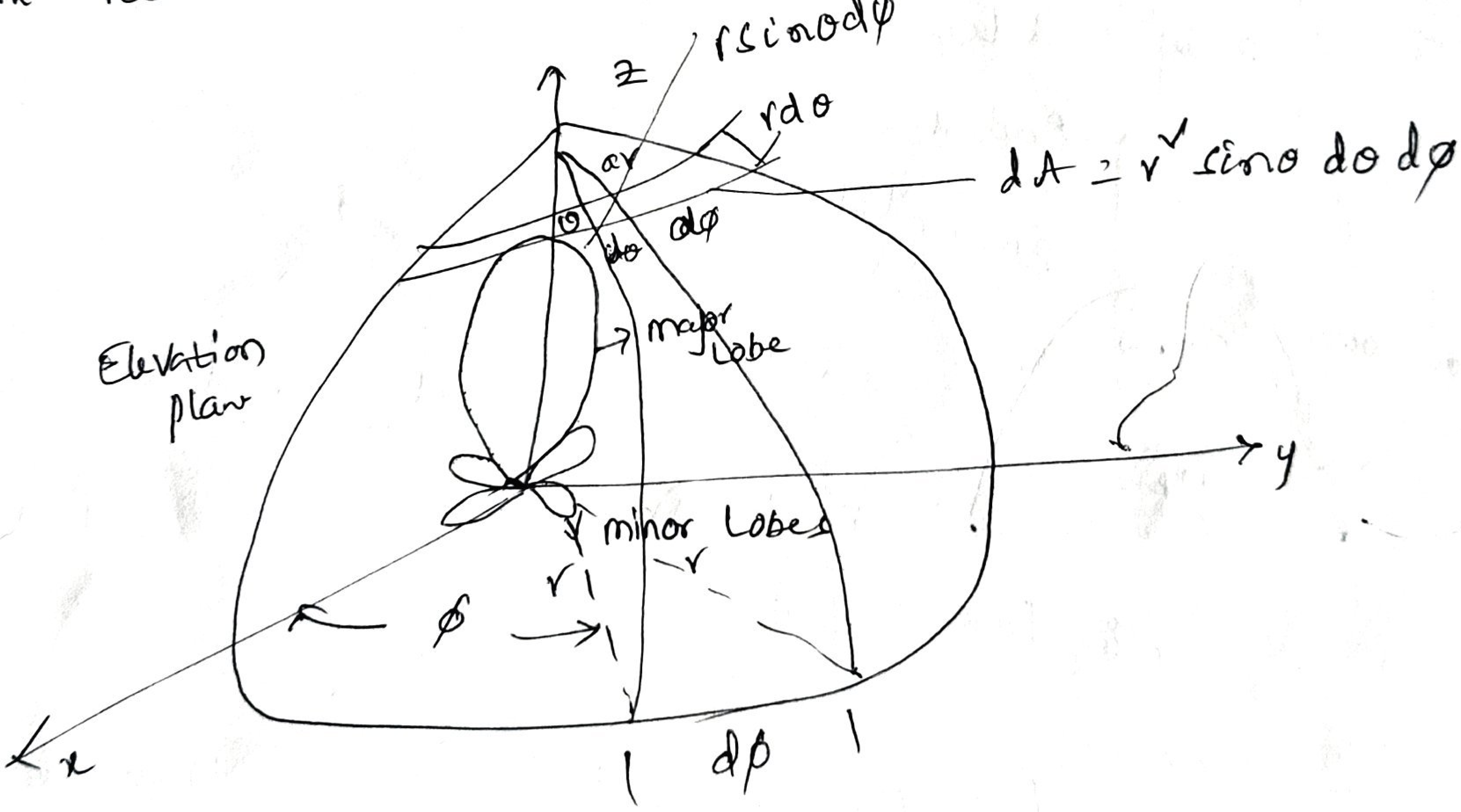


fig: Coordinate System for antenna analysis

a) Isotropic, Directional and Omnidirectional patterns:

Isotropic → Equal radiation in all direction.

Directional :- Having more radiation in some directions than others

Omni → Special types of directional patterns.

b) Principle Pattern.

for linearly polarized antenna E and H planes are there

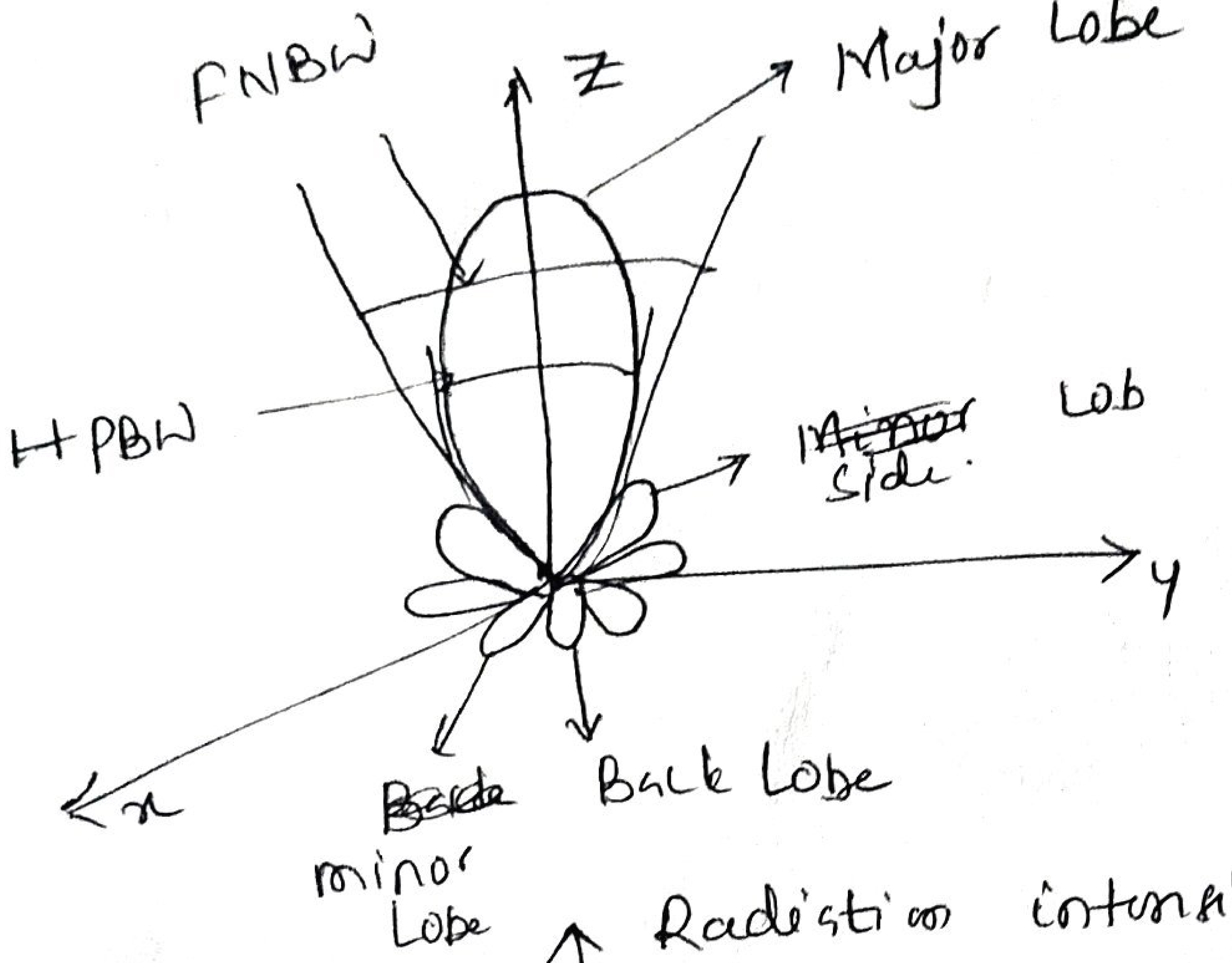
E → Electric field Vector

H → Magnetic field ,,

for x-z plane with elevation $\phi = 0$ it is E-plane

,, x-y ,, with azimuth plane $\theta = \pi/2$ H-plane.

c) Radiation Pattern Lobes:



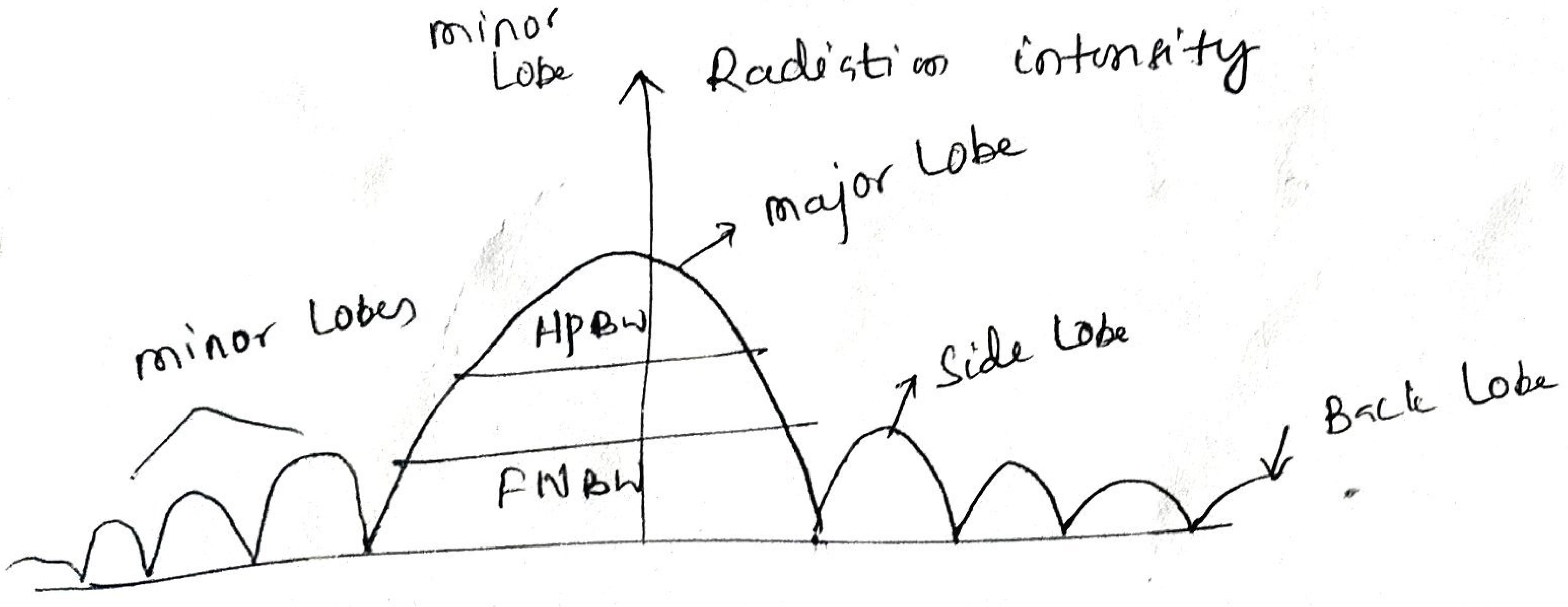
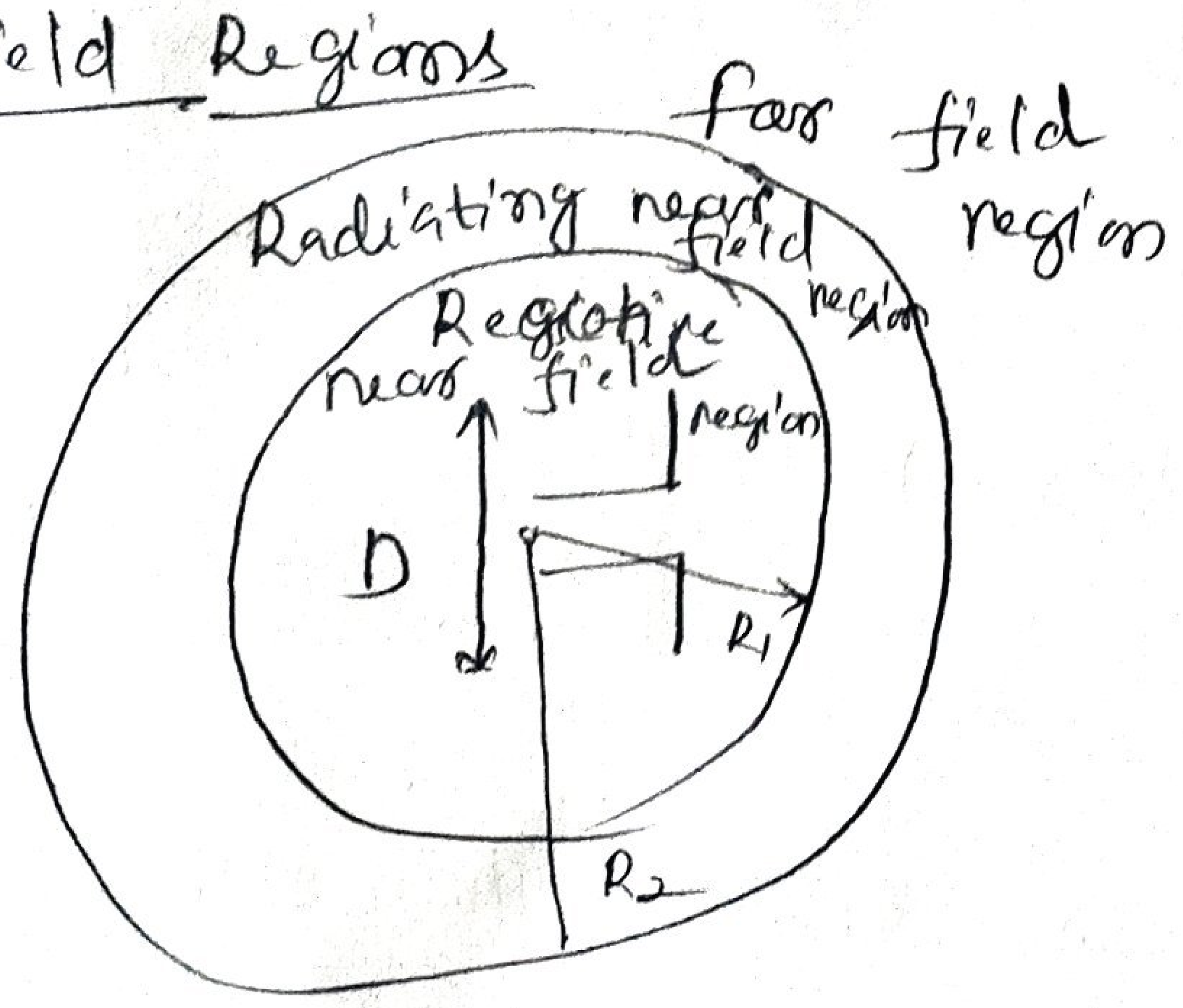


fig: Power Pattern.

D) field Regions

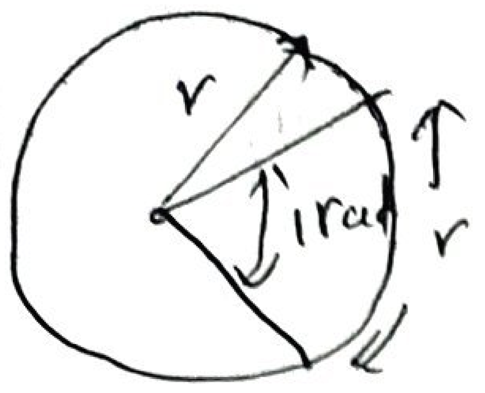


D → Largest dimension of antenna

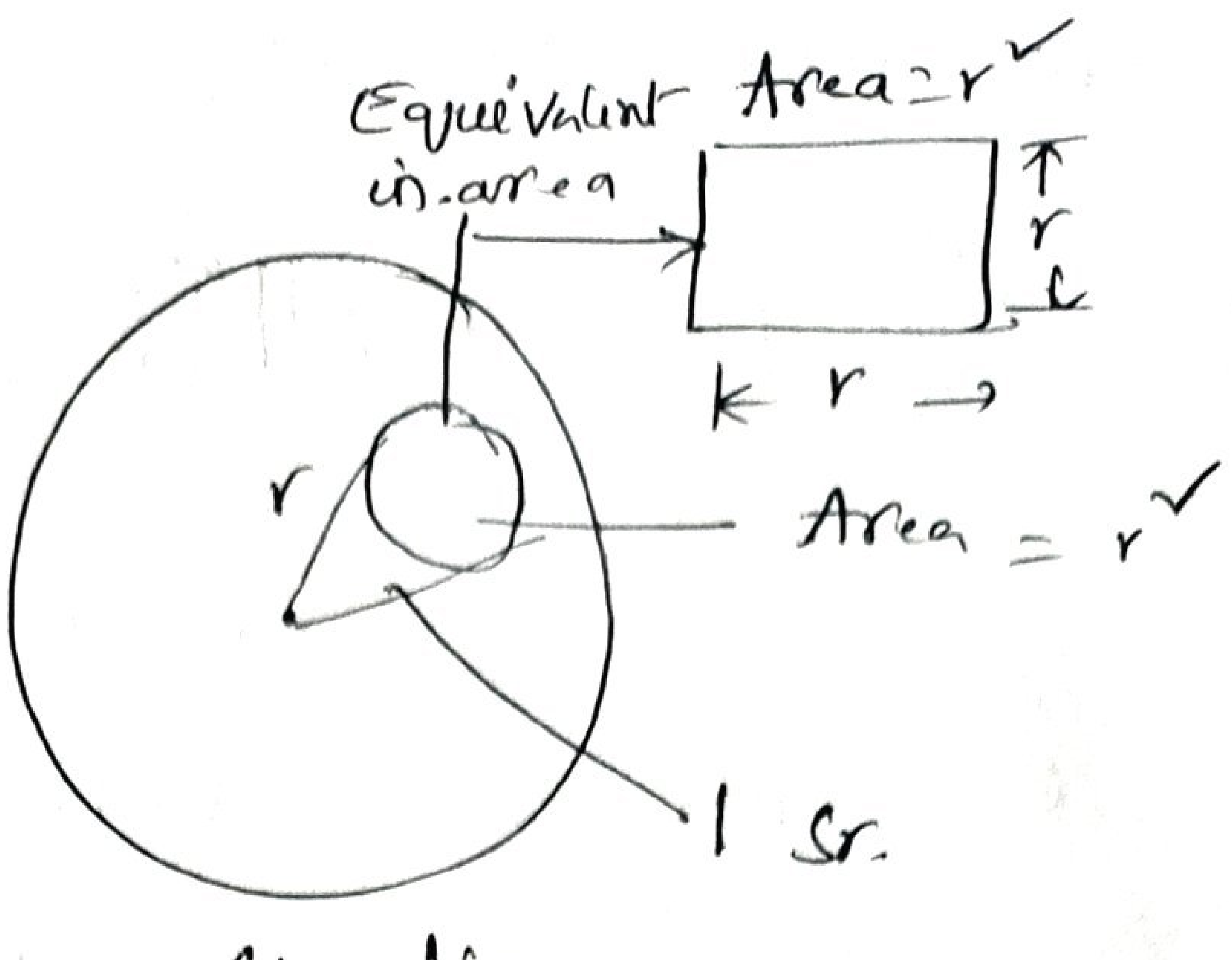
$R_1 = 0.62\sqrt{D^3/\lambda}$

$R_2 = 2D^2/\lambda$

e) Radian and Steradian



Radian



Steradian

The measure of Plane angle is Called radian. It is defined as the plane angle with Vertex at the centre of a Circle with radius r. i.e subtended by an arc.

$$C = 2\pi r.$$

→ The measure of solid angle is Steradian. One steradian is defined as the solid angle with vertex at the centre of the sphere with radius r, i.e subtended by surface area equal to that square with side lengths r. $A = 4\pi r^2$

Infinitesimal area $dA = r^2 \sin\theta \, d\theta \, d\phi$ m^2

∴ Element of solid angle $d\Omega = \dfrac{dA}{r^2} = \sin\theta \, d\theta \, d\phi$ Sr.

2. Radiation Intensity:

Radiation intensity in given direction is defined as "the power radiated from unit Solid angle. It is a far field parameter and can be obtained by simply multiplying the radiation density by the square of the distance.

$$U = r^2 W_{rad}.$$

Where U = radiation intensity W/unit solid angle

W_{rad} = radiation density (W/m^2)

It is also related to far - zone electric field

$$U(\theta,\phi) = \frac{r^2}{2\eta}\left|E(r,\theta,\phi)\right|^2 \simeq \frac{r^2}{2\eta}\left[\left|E_\theta(r,\theta,\phi)\right|^2 + \left|E_\phi(r,\theta,\phi)\right|^2\right]$$

$\ni$

Where E = far - zone electric field intensity of the antenna

E_θ, E_ϕ = far - zone electric field components of "

η = Intrinsic impedance of the medium.

$$P_{rad} = \oiint_\Omega U\, d\Omega = \int_0^{2\pi}\int_0^{\pi} U \sin\theta \cdot d\theta \cdot d\phi$$

Where $d\Omega$ = element of solid angle $= \sin\theta\, d\theta \cdot d\phi$.

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Beam Solid Angle or Beam Area:-

In polar two dimensional coordinates as incremental area dA on the surface of a sphere is the product of the length rdθ in the θ direction and r sinθ dφ in the φ direction i.e Latitude and Longitude.

$$dA = r d\theta \cdot r\sin\theta \cdot d\phi$$
$$= r^2 \sin\theta \cdot d\theta \cdot d\phi$$
$$= r^2 \, d\Omega$$

where dA is solid angle expressed in steradian

dA is solid angle subtended by antenna

$$\text{Area of the sphere} = 2\pi r \sin\theta \cdot r d\theta$$
$$= 2\pi r^2 \sin\theta \cdot d\theta$$
$$= 2\pi r^2 \int_0^{\pi} \sin\theta \cdot d\theta = -\cos\theta \Big|_0^{\pi}$$
$$= 4\pi r^2$$

Where $4\pi r^2$ is the solid angle subtended by sphere.

Now beam solid angle or beam are for power pattern is

$$\Omega_A = \int_{\phi=0}^{\phi=2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta \cdot d\theta \cdot d\phi$$

$$= \iint_{4\pi} P_n(\theta, \phi) \, d\Omega$$

$$\text{Beam area} = \Omega_A = \phi_{HP} \cdot \theta_{HP}$$

It is interms of angles subtended by half power points of the main lobe in the two principle patterns.

Directivity (D), Directive Gain $G_D(\theta,\phi)$:

Directivity (D): It is the ratio of the radiation intensity to average radiation intensity in all directions.

$$D = \frac{U}{U_0} = \frac{U}{U_{avg}}$$

Directive Gain $G_D(\theta,\phi)$: It is a measure of the concentration of the radiated power in a particular direction θ, ϕ

It is the maximum power density to average power radiated is called maximum directive gain or directivity of the antenna. It is denoted by $G_{D\,max}$.

$$G_D(\theta,\phi) = \frac{P_d(\theta,\phi)}{P_{avg}} = \frac{P_d(\theta,\phi)}{\frac{P_{rad}}{4\pi r^2}} = \frac{P_d(\theta,\phi)\cdot r^2}{\frac{P_{rad}}{4\pi}}$$

$$= \frac{U(\theta,\phi)}{U_{avg}} = \frac{4\pi\, U(\theta,\phi)}{P_{rad}}$$

$$\therefore D = G_{D\,max} = \frac{P_{d\,max}}{\frac{P_{rad}}{4\pi r^2}}$$

$$= \frac{U_{max}}{U_{avg}} = \frac{4\pi\, U_{max}}{P_{rad}}.$$

→ The directivity of an antenna is dimensionless quantity.

The directivity can also be expressed in terms of the electric field intensity as

$$D = G_{D\,max} = \frac{4\pi\, |E_{max}|^2}{\int_0^{2\pi}\int_0^{\pi} |E(\theta,\phi)|^2 \sin\theta \cdot d\theta \cdot d\phi}$$

Antenna Gain:-

It is ratio of radiation intensity in given direction to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

$$\text{Gain} = 4\pi \cdot \frac{\text{radiation intensity}}{\text{total input power}}$$

$$= 4\pi \cdot \frac{U(\theta,\phi)}{P_{in}}$$

$$= 4\pi \cdot \frac{U(\theta,\phi)}{P_{in}}$$ (Loss less isotropic source)

$$P_{rad} = e_{cd} \cdot P_{in}$$

Where e_{cd} = antenna radiation effeciency

$$G(\theta,\phi) = e_{cd}\left[\frac{4\pi\, U(\theta,\phi)}{P_{rad}}\right]$$

$$= e_{cd}\, D(\theta,\phi)$$

$$G_0 = G(\theta,\phi)_{max} = e_{cd}\, D(\theta,\phi)_{max}$$

$$= e_{cd}\, D_0$$

$$G_0 = G_\theta + G_\phi$$

$$G_\theta = \frac{4\pi U_\theta}{P_{in}}$$

$$G_\phi = \frac{4\pi U_\phi}{P_{in}}$$

Where U_θ = radiation intensity in given direction contained in θ field components.

U_ϕ = radiation intensity in given direction contained in ϕ field components

P_{in} = Total input / accepted power.

Polarization:-

It is the orientation of electromagnetic signals in the free space. It has time varying direction and relative magnitude of the electric field vector.

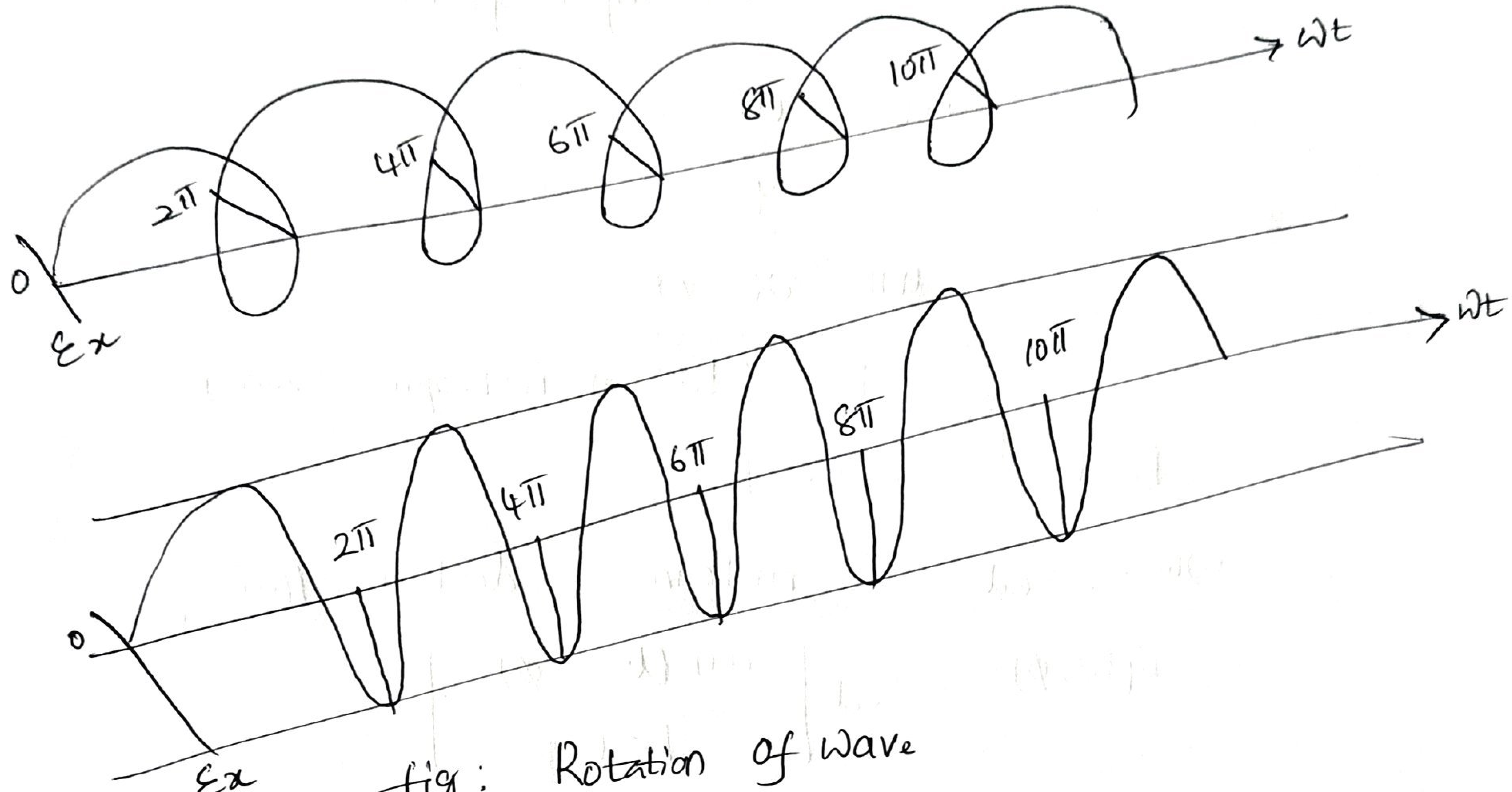

fig: Rotation of wave

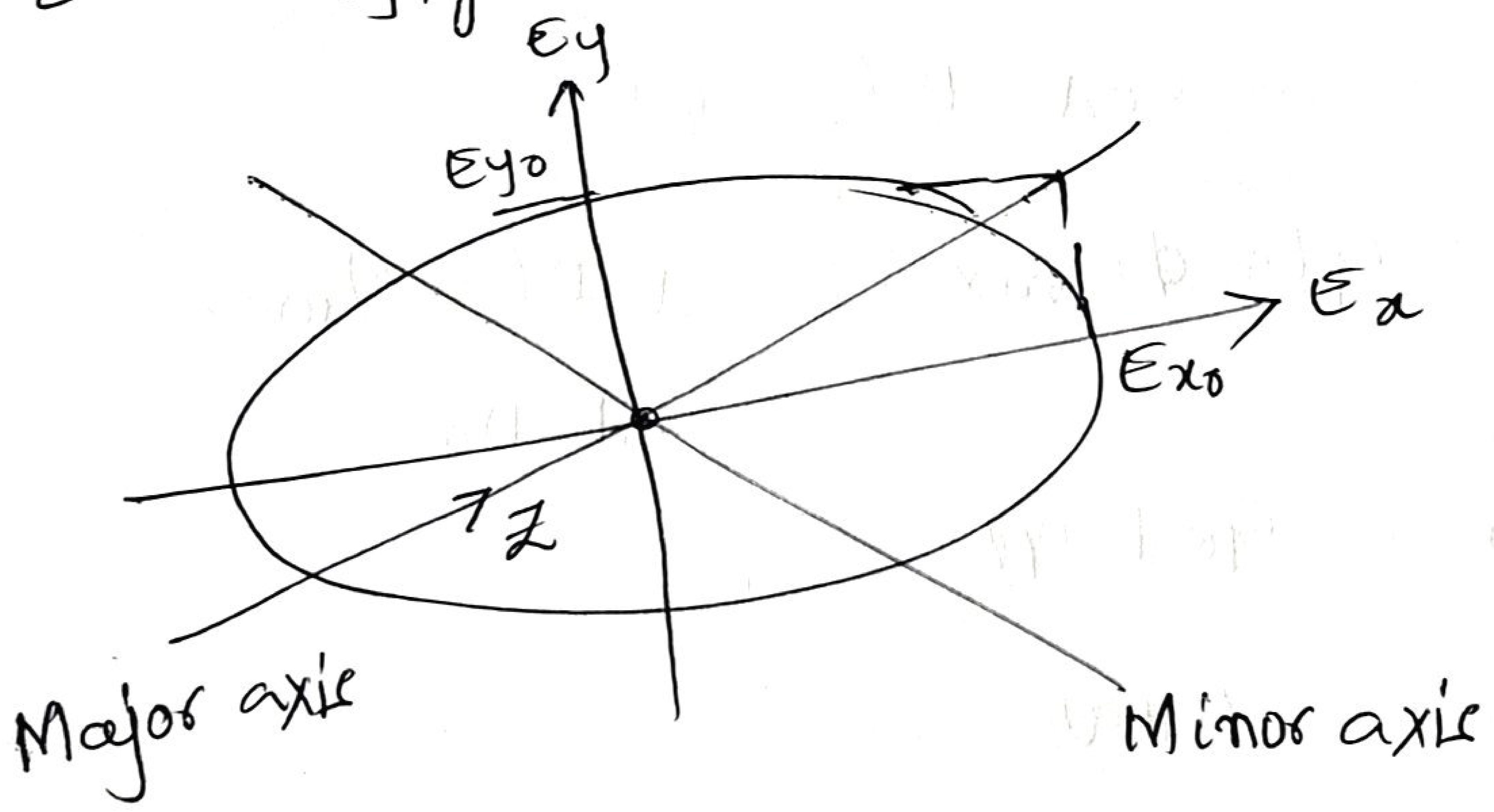

fig: Elliptical polarization

The polarization is linear, circular and elliptical.

Linear Polarization:-

Electric field at a point in a space as a function of time as always along line. So, polarization equations along negative Z - direction.

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$$\mathcal{E}(z;t) = \hat{a}_x \mathcal{E}_x(z;t) + \hat{a}_y \mathcal{E}_y(z;t)$$

$$\mathcal{E}_x(z;t) = Re\left[\mathcal{E}_x e^{-j(\omega t + kz)}\right]$$

$$= Re\left[\mathcal{E}_{x0} e^{j(\omega t + kz + \phi x)}\right]$$

$$= \mathcal{E}_{x0} \cos(\omega t + kz + \phi x)$$

$$\mathcal{E}_y(z;t) = Re\left[\mathcal{E}_y e^{-j(\omega t + kz)}\right]$$

$$= Re\left[\mathcal{E}_{y0} e^{j(\omega t + kz + \phi y)}\right]$$

$$= \mathcal{E}_{y0} \cos(\omega t + kz + \phi y)$$

$\mathcal{E}_{x0}$ and $\mathcal{E}_{y0}$ are the magnitudes of x and y components

so $\Delta\phi = \phi_y - \phi_x = n\pi, \quad n = 0, 1, 2, 3, \ldots$

a) field components are only component

b) Two orthogonal Linear components that are in time phase or $180°$ out of phase.

Circular Polarization :-

Magnitudes of two components are same or the time phase difference them is odd multiples of $\frac{\pi}{2}$.

$$|\mathcal{E}_x| = |\mathcal{E}_y| \Rightarrow \mathcal{E}_{x0} = \mathcal{E}_{y0}$$

$$\Delta\phi = \phi_y - \phi_x = \begin{cases} +\left(\frac{1}{2} + 2n\right)\pi, & n = 0, 1, 2, \ldots \text{ for C.W} \\ -\left(\frac{1}{2} + 2n\right)\pi, & n = 0, 1, 2, \ldots \text{ for CCW} \end{cases}$$

a). The field must have two orthogonal linear components

b) Two components must have same magnitude

c) Two components must have time phase difference of odd multiples.

Elliptical Polarization :-

It is occurs time-phase difference between two components is odd multiples of $\frac{\pi}{2}$.

$|\mathcal{E}_x| \neq |\mathcal{E}_y| \Rightarrow \mathcal{E}_{x0} \neq \mathcal{E}_{y0}$

When $\Delta\phi = \phi_y - \phi_x = \begin{cases} +(\frac{1}{2}+2n)\pi & \text{for CW} \\ -(\frac{1}{2}+2n)\pi & \text{for CCW} \end{cases}$

$n = 0, 1, 2, \ldots$

or

$\Delta\phi = \phi_y - \phi_x = \pm\frac{n}{2}\pi = \begin{cases} >0 & \text{for CW} \\ <0 & \text{for CCW} \end{cases}$

$n = 0, 1, 2, \ldots$

Axial Ratio. $A.R = \dfrac{\text{Major axis}}{\text{Minor axis}} = \dfrac{OA}{OB}, \quad 1 \leq AR \leq \infty.$

a) Field must have two orthogonal linear

b) Two components can be of same magnitude or different magnitude.

c) Two components are not of the same magnitude $0°$ or multiples of $180°$. If the two components are of the same magnitude, the time phase difference between two components must not be odd multiples of $\frac{\pi}{2}$.

<u>Antenna Effeciency:-</u>

The overall antenna effeciency $e_0 = e_r e_c e_d$

Where e_0 = Total effeciency

e_r = Reflection effeciency $(1 - |\Gamma|^2) \rightarrow$ this is mismatch condition

e_d = Dielectric effeciency

Γ = Voltage reflection coefficient

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

Usually e_c and e_d are very difficult to measure.

$\therefore e_0 = e_r e_{cd} (1 - |\Gamma|^2)$

Equivalent Areas:-

With each antenna, we can a number of equivalent area. These are used to describe power capturing characteristics of the antenna when a wave impenging on it. One of these equivalent area is the effective area. Which in a given direction it is defined as the ratio of the availabe power at the terminals of a receiving antenna to the power flux density of a plane wave incident on the antenna from that direction, the wave being polarization matched to the antenna. If direction is not specified, the direction of maximum radiation intensity is implied.

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 \frac{R_T}{2}}{W_i}$$

Where

A_e = Effective Area (m^2)

P_T = Power delivered to the Load (ω)

W_i = Power density of incident wave (ω/m^2)

$$A_e = \frac{|V_T|^2}{2W_i}\left[\frac{R_T}{(R_r + R_L + R_T)^2 + (X_A + X_T)^2}\right]$$

Maximum effective aperture

$$A_{em} = \frac{|V_T|^2}{8W_i}\left[\frac{R_r}{(R_L + R_r)^2}\right] = \frac{|V_T|^2}{8W_i} \cdot \frac{1}{R_L + R_r}$$

The different effective areas are

Scattering area $A_s = \dfrac{|V_T|^2}{8W_i}\left[\dfrac{R_r}{(R_L+R_r)^2}\right]$

Loss area $A_L = \dfrac{|V_T|^2}{8W_i}\left[\dfrac{R_L}{(R_L+R_r)^2}\right]$

Capture Area $A_C = \frac{|V_T|^2}{8W_i}\left[\frac{R_T+R_L+R_r}{(R_L+R_r)^2}\right]$

Capture area = Effective area + Scattering area + Loss area

Aperture effeciency $\epsilon_{ap} = \frac{A_{em}}{A_p} = \frac{\text{Maximum effective area}}{\text{Physical area.}}$

Radiation Resistance : (R_{rad})

It is the part of an antenna's feed point, electrical resistance caused by the emission of radio waves from the antenna. It is related to input impedance of antenna.

$$Z_A = R_A + jX_A$$

Where Z_A = antenna impedance

R_A = antenna resistance

X_A = antenna reactance

$$R_A = R_r + R_L$$

where R_r = Radiation resistance

R_L = Loss resistance.

$$P_d' = I^2 R$$

$$R = \frac{P_d'}{I^2}$$

$$P_d = P_d' + P_d''$$

= Ohmic Loss + Radiation Loss

$$P_d = I^2 R_{rad} + I^2 R_{Loss}$$

$$= I^2 (R_{rad} + R_{Loss})$$

The Effective length of the antenna:—

The effective length of the antenna, whether it be linear or an aperture antenna is a quantity that is used to determine the voltage induced on the open-circuit termination of antenna when a wave impinges upon it. The vector effective length l_e for an antenna is usually a complex vector quantity represented by

$$l_e(\theta, \phi) = \hat{a}_\theta \, l_\theta(\theta, \phi) + \hat{a}_\phi \, l_\phi(\theta, \phi)$$

This is also referred to as effective height. It is a far field quantity and it is related to the far-zone field E_a radiated by the antenna, with current I_{in}.

$$E_a = \hat{a}_\theta E_\theta + \hat{a}_\phi E_\phi = -j\eta \frac{k \cdot I_{in}}{4\pi r} l_e e^{-jkr}.$$

The effective length represents the antenna in its transmitting and receiving modes, and it is particularly useful in relating the open circuit voltage V_{oc} of receiving antenna. The relation can be expressed as

$$V_{oc} = E^i \cdot l_e$$

Where

V_{oc} = open circuit voltage at antenna terminals

E^i = Incident electric field

l_e = Vector effective length

when l_e and E^i are linearly polarized

$$E^i = l_e.$$

Antenna Temperature:-

For every object physical temperature is 0 K or −273°c

The amount of energy radiated is usually represented by an equivalent temperature T_B; known as brightness temperature.

$$T_B(\theta, \phi) = \epsilon(\theta, \phi)\, T_m = (1 - |\Gamma|^2)\, T_m$$

Where T_B = Brightness temperature (equivalent temperature; K)

ϵ = Emissivity (unit less)

T_m = Molecular temperature (physical temperature)

$\Gamma(\theta, \phi)$ = reflection Coefficient of the surface for the polarization of the wave.

The value of emissivity are $0 \leq \epsilon \leq 1$

The brightness temperature emitted by different sources is intercepted by antennas, and it appears at their terminals as an antenna temperature. In equation form it can be written as

$$T_A = \frac{\int_0^{2\pi}\int_0^{\pi} T_B(\theta, \phi)\, G(\theta, \phi) \sin\theta \cdot d\theta \cdot d\phi}{\int_0^{2\pi}\int_0^{\pi} G(\theta, \phi) \sin\theta \, d\theta \, d\phi}$$

Where T_A = Antenna Temperature

$G(\theta, \phi)$ = Gain pattern of the Antenna

Assuming no losses or other contributions between the antenna and receiver, the noise power transferred to the receiver is given by

$$P_r = k T_A \Delta f$$

Where P_r = antenna noise power (W)

k = Boltzman's constant (1.38×10^{-23} J/K)

T_A = Antenna temperature (k)

Δf = Bandwidth (Hz)

Relation between Maximum Directivity and Maximum Effective Area:

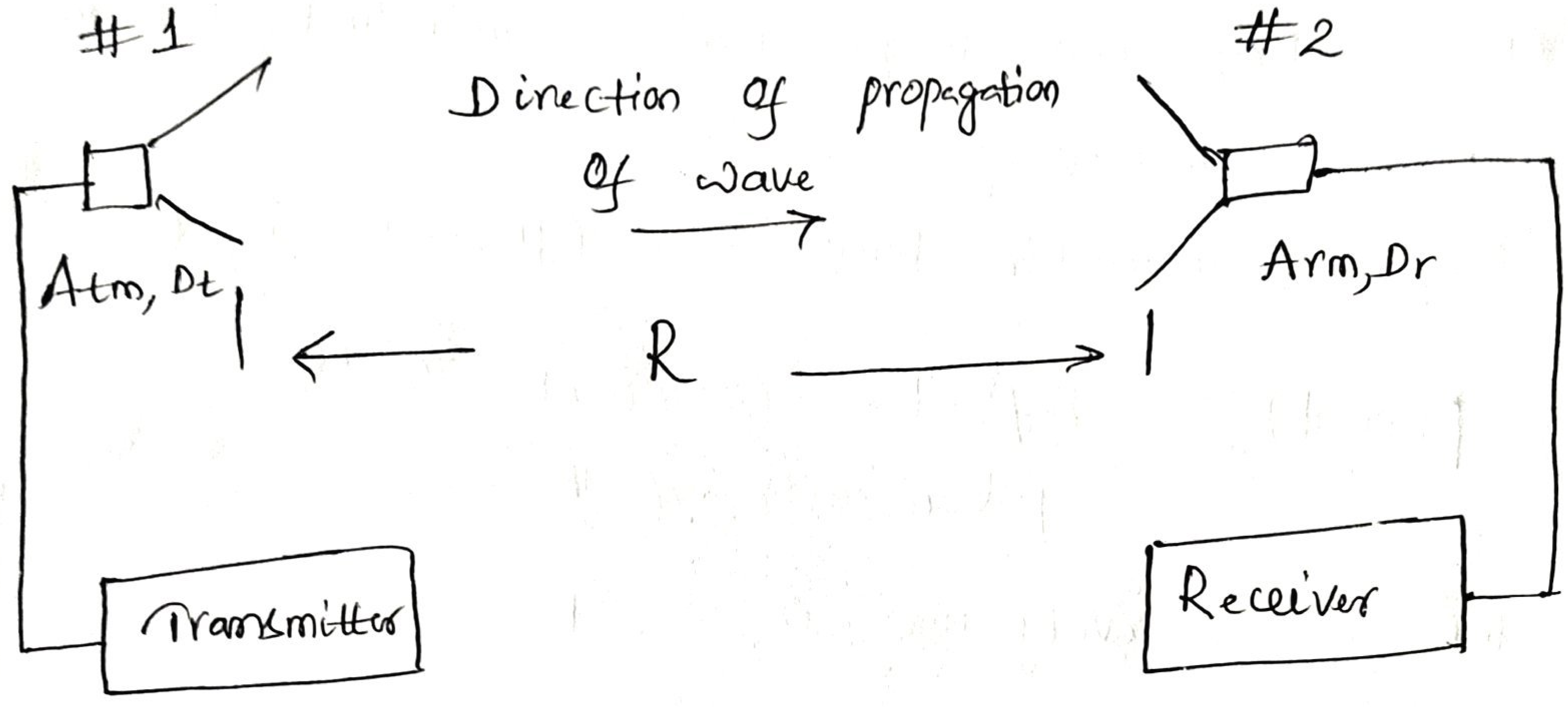

fig: Two antennas separated by a distance R.

To derive the relationship between directivity and maximum effective area, the geometrical arrangement of above figure is chosen. Antenna 1 is used as a transmitter and 2 is used as a receiver. The effective area and directivities of each are designed as A_t, A_r and D_t, D_r. If antenna 1 were isotropic, its radiated power density at a distance R would be

$$W_0 = \frac{P_t}{4\pi R^2}$$

where P_t is the total radiated power.

$$W_t = W_0 D_t = \frac{P_t D_t}{4\pi R^2}$$

The power collected (received) by the antenna and transformed to the load would be

$$P_r = W_r A_r = \frac{P_t D_t A_r}{4\pi R^2}$$

or $$D_t A_r = \frac{P_r}{P_t}(4\pi R^2) \quad \text{— (a)}$$

If antenna 2 is used as transmitter, 1 as a receiver, and the intervening medium as linear, Passive and isotropic we can write that

$$D_r A_t = \frac{P_r}{P_t}(4\pi R^2) \quad \text{— (b)}$$

Equating a & b, reduced to

$$\frac{D_t}{A_t} = \frac{D_r}{A_r}$$

Increasing the directivity of an antenna increases its effective area in direct proportion. Thus

$$\frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}}$$

Where A_{tm} and A_{rm} are maximum effective areas and D_{ot}, D_{or} are directivities of antennas 1 and 2 respectively.

If antenna 1 is isotropic, then $D_{ot} = 1$ and its maximum effective area can be expressed as

$$A_{tm} = \frac{A_{rm}}{D_{or}}$$

$$= \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}$$

$$A_{rm} = A_{tm} D_{or} = D_{or}\left(\frac{\lambda^2}{4\pi}\right)$$

In general, the maximum effective aperture (A_{em}) of any antenna is related to its maximum directivity (D_0) by

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

$$A_{em} = e_t\left(\frac{\lambda^2}{4\pi}\right) D_0 \, |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$= e_{cd}(1 - |\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right) D_0 \, |\hat{\rho}_w \cdot \hat{\rho}_a|^2.$$

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