PRESTRESSED CONCRETE

SYLLABUS:

UNIT-1:

Basic concept of Prestressing - Advantages and application of Prestressed concrete, High Strength concrete - Permissible stresses, shrinkage, creep, Deformation charcterstics, High strength steel - Types, strength - Permisible stresses - Relaxation of stress, cover requirments.

UNIT-2:

Prestressing systems - Introduction, Tensioning devices, Pretensioning systems, post tensioning systems, Basic assumptions in analysis of Prestress and design, Analysis of Prestress, Resultant stresses at a section - pressure line - concepts of load balancing - stresses in Tendons, cracking moment.

UNIT-3:

Losses of Pre-stressing - Loss of prestress in Pre-tensioned and Post tensioned member due to various causes - Elastic Shortening of concrete, Shrinkage of concrete, creep of concrete, Relaxation of stress in steel, slip in anchorage, differential Shrinkage - bending of members and frictional losses - Total losses allowed for design.

UNIT-4:

Design for flexural resistance - types of flexural failure - code procedures, design of section for flexure. control of deflection - Factor influencing deflection - Prediction of short term and long term deflection.

UNIT 5:

Design for Shear & Tension- shear & principal stresses- Design of shear reinforcements - codal provisions- Design for torsional, Design for combined bending- Shear & torsion.

UNIT 6:

Transfer of prestress in pre tensioned members- transmission length - Bond stresses - end zone reinforcement - codal provisions- Anchorage zone stresses in post tensioned members - stress distribution in end block - Anchorage zone reinforcement.

BASIC CONCEPT OF PRE-STRESSED CONCRETE :

Pre-Stressed concrete is basically a concrete in which internal stresses of a Suitable magnitude and distribution are introduced, so that the stresses resulting from the external loads (or) concrated to a desired degree.

* Reinforced concrete commonly introduced by tensioning the steel reinforcement.

Ex: Example of wooden barrel construction by force fitting of metal bands and Shrink fitting of metal tyres. on wooden wheels. indicate that the art of Prestressing. as been practied from ancient times.

* The development of earlier cracks in reinforced concrete due to incompactability in the strain of steel. and concrete was perhaps the starting point in the development of new material like pre-stressed concrete.

Freyssinet :

In 1904, Freyssinet attempted to introduce Permanetly acting forces in concrete to resist the elastic forces developed under loads and these idea was later developed under the name of "Pre-stressed."

Cracks will occur in Reinforced concrete deformation will occur in Prestressed concrete.

Advantages:

* Members are free from the tensile stress.
* High ability to resist the impact.
* Fatigue resistance is high.
* High live load carrying capacity.
* No crack formation.
* Better corrossion resistance.
* very effective for deflection control.
* Need less material.

Disadvantages:

* more expensive
* more complex technology.
* Hard to recycle.
* Need higher quality materials.

Applications:

During last 60 years pre-stressed concrete has been widely used for long & the construction of long span bridges, slabs, tanks, concrete pile, thin shell structures, off shore platforms, nuclear power plant repair and rehabitation

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High Strength Concrete:

For high Strength concrete:

* 28 days → fck 30-70N/mm².

Low Shrinkage minimum creep charcterstics and high value of young's modouls are generally deemed necessary for concrete used for pre stressed members.

Recent days ultra high strength forme concrete formed as increased from fck → 70-100N/mm².

Strength Requirments:

minimum 28 days fck IS: 1343

for Pretensioned - 40N/mm²

for Post tensioned - 30N/mm²

Permissible Stresses in concrete:

Indian standard code, Permissible compressive stress in flexure varies from 0.41 for m_{30} grade concrete to a value of 0.35 for m60 grade concrete.

Shrinkage of concrete:

It is due to the gradual loss of moisture which results in change in volume.

Is code for purpose of design for

Pre / Post - tensioned member $\frac{2 \times 10^{-4}}{\log_{10}(T+2)}$.

$\therefore T =$ age in days.

Post / Pre tensioned member 3×10^{-4}

<u>Creep of concrete:</u>

→ Deformation due to externally applied stresses generally refered to as a creep.

→ Deformation which occurs without any externally stresses refred to as a shrinkage

→ Rate of creep decreases with time.

→ 55% to of 20 years creep occurs in 3 months.

→ 77% of 20 years creep occurs in 1 year. After 1 year load is taken as unity.

→ The average value of creep at later age - 1.26 after 10 years.
1.36 after 30 years.

→ As IS : 1343 creep coefficient (C_c)

$$C_c = \frac{\text{ultimate creep strain}}{\text{elastic strain}}$$

Creep values:

C_c - 2.2 for 7 days.

C_c - 1.6 for 28 days

C_c - 1.1 for 1 year

Deformation characteristics of concrete:

$E = 5000\sqrt{F_{ck}}$. (∴ IS 1343).

High Strength Steel:

High Strength Steel (H.S.S) is generally acheived by increasing the carbon content compare to mild steel.

0.6 - 0.85% carbon

0.7 - 1% Magnesium

0.05% Sulphur & phosporow.

High tensile steel bars commonly used in Pre stressing manufacturing in nomial sizes of 10, 12, 16, 20, 22, 25, 28 and 32 mm diameter.

Types:

1. Wires - single unit of steel.
2. Stands - two/three/seven wires are wound
3. Tendons - Group of stands/wire
4. Cables - Group of tendons.
5. Bars. - A tendon can be made up of a single steel bar. The diameter of bar is increase.

Permissible stress in Steel

$$\frac{\text{ultimate strength}}{\text{yield strength}}$$

Relaxation of stress:

Decreasing of stress in steel at constant strain.

Stress Corrossion & cover Requirment

In the duct of post tensioned members are not grouted their is a possibility of stress corrossion leading to a failure of the structure.

Some of the important protective measurments against stress corrossion include protection from chemical contamination protective coatings for high tensile steel and grouting of ducts immediately after Pre-stressing operations.

Cover Requirments

As per IS 1343 - Pre tensioned minimum clean cover is 20mm

- Post tensioned 30mm (or) size of cable. (taken which ever is greater).

If Pre stressed member are exposed to aggressive environment cover requirment is increased to 10mm.

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UNIT-3

Losses of Prestressing

Pretension:

1. Elastic deformation
2. Shrinkage of concrete.
3. Creep of concrete.

Post tensioned.

1. No elastic deformation due to simultaneously tensioned & successive tensioned elastic deformation occur.
2. Shrinkage of concrete
3. Creep of concrete.
4. Friction.
5. Anchorage slip.

Losses due to elastic deformation:

$E_e = \frac{f_c}{E_c}$, strain = $\frac{\text{force (stress)}}{\text{young's modulus}}$

$= \frac{f_c}{E_c} E_s$

Es = young's modulus of steel
Ec = young's modulus of concrete
fc = Prestressing force.

modular ratio = $\left[\frac{E_s}{E_c}\right] f_c$

Loss due to elastic deformation $= \alpha_e f_c$.

Losses due to shrinkage: It is due to shortening of wires.

tensioned Prestressing $= 300 \times 10^{-6}$

Post tensioned $= \frac{200 \times 10^{-6}}{\log_{10}(T+2)}$

$\Big\} \varepsilon_{cs}$. (Pg:16)

Loss due to shrinkage $= \varepsilon_{cs} \times E_s$

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Loss due to creep:

Deformation due to sustained load is called as creep.

1) ultimate creep strain method $= \varepsilon_{cc} * f_c E_s$.

2) Creep coefficient method $(\phi) = \frac{\text{creep strain}}{\text{Elastic strain}} = \frac{\varepsilon_c}{\varepsilon_e}$

$$\varepsilon_c = \phi . \varepsilon_e$$

$$\varepsilon_c = \phi \left(\frac{f_c}{E_c} \right) E_s$$

$$\varepsilon_c = \phi \left(\frac{E_s}{E_c} \right) f_c$$

$$= \phi \alpha_e f_c$$

Loss due relaxation of steel:

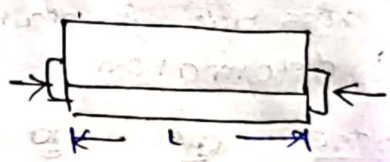
decrease stress with time under constant strain

$\underbrace{0.5 f_{pu} \text{ to } 0.85 f_{pu}}_{\text{Initial stresses}}$. varies $\underbrace{0 - 90 \ N/mm^2}_{\text{Relaxation losses}}$.

Loss due to Anchorage slip: (post tension)

$$\Delta = \frac{PL}{A E_s}$$

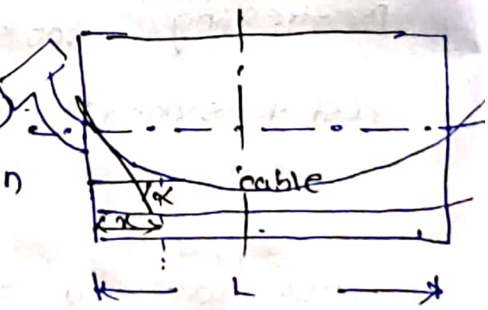
$$\frac{P}{A} = \frac{E_s \Delta}{L}$$



Loss due to friction: (post tension)

$$P_x = P_0 e^{-(\mu \alpha + K x)}$$

(Pg: 33)



where

μ = coefficient friction in curve.

K = coefficient for wave effect.

α = cumulative angle.

P_0 = Prestressing force at the tensioning end.

Elastic deformation problems:

1. A Pre tensioned concrete beam of rectangular c/s section 150mm wide & 300mm deep is Pre-stressed by 8 tensile wires of 7mm ϕ are located at 100mm soffit of the beam. If the wires are tensioned to a stress of 1100 N/mm^2 calculate the percentage loss of stress due to elastic deformation. Assume the modolues of elasticity of concrete (Ec) and steel (Es) as 31.5 & 210 N/mm^2

Sol: Given data.

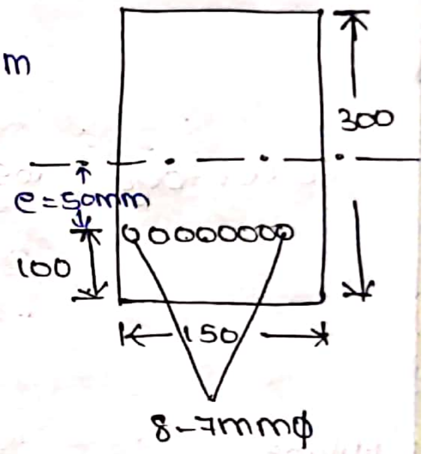
c/s section = 150mm * 300mm

c/s Area = 45000 mm^2

Stress = 1100 N/mm^2

E_c = 31.5 N/mm^2

E_s = 810 N/mm^2



$$\text{force}(F) = \frac{\text{Load}(P)}{\text{Area}(A)}$$

Stress (F) * Area (A) = Load(P)

$1100 * 45*10^3$

Area of steel bars = $n \cdot \frac{\pi}{4}(d)^2$

$= 8 \cdot \frac{\pi}{4}(7)^2$

$= 307.87$ mm^2

Load(P) = $1100 * 307.87$.

P = $338.66 * 10^3$ N

Stress at level of steel $(f_c) = \frac{P}{A} + \frac{Pe^2}{I}$

f_c = 338.66.

$$I = \frac{bd^3}{12} = \frac{(150)(300)^3}{12}$$

$I = 337.5 * 10^6$ mm^4

$$f_c = \frac{338.66 \times 10^3}{45 \times 10^3} + \frac{338.66 \times 10^3 \times (50)^2}{337.5 \times 10^6}$$

$$= 7.52 + 2.5$$

$$\boxed{f_c = 10\ N/mm^2}$$

Loss of stress due to elastic deformation of concrete $= \alpha_e f_c$

$$= \left(\frac{E_s}{E_c}\right) f_c$$

$$= \left(\frac{210}{31.5}\right) \times 10$$

$$= 66.66\ N/mm^2$$

Percentage loss of stress in steel

$$= \frac{66.66}{1100} \times 100$$

$$= 6.06\%$$

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2) A post tensioned concrete beam 100mm wide, 300mm deep is Pre Stressed by 3 cables each with a c/s sectional area of $50mm^2$ and with initial stresses of $1200\ N/mm^2$. All the 3 cables are straight and located 100mm from the soffit of the beam. If the modular ratio is 6. calculate the loss of stress in the 3 cables due to elastic deformation of concrete for only the following cases

a) simultaneously tensioned & anchoring of all the three cables.

b) Successive tensioning of the 3 cables, one at a time.

Sol : Given data

wide(b) = 100mm

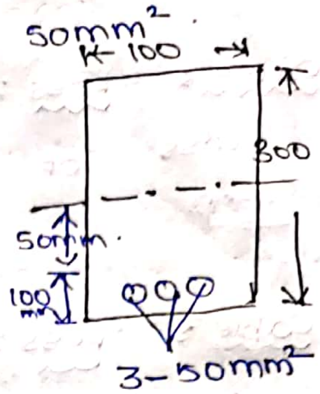
deep(d) = 300mm.

3 cables with c/s area 50mm^2.

Intial stress = 1200 N/mm^2.

modolus ratio(α_e) = 6. $\left(\frac{E_s}{E_c}\right)$.

eccentricity(e) = 50mm.



area of the beam = 100 * 300

= $3*10^4$ mm^2.

$$\text{Stress}(F) = \frac{\text{load}(P)}{\text{Area}}$$

P = Stress * Area

Area = ~~n×Ø~~ 50mm^2

P = 1200 * 50

P = $60*10^3$ N

$\boxed{P = 60\,KN}$

Stresses at the level of steel $(F_c) = \frac{P}{A} + \frac{Pe^2}{I}$

$$I = \frac{bd^3}{12} = \frac{(100)(300)^3}{12} = 225*10^6\ mm^4$$

$$F_c = \frac{60*10^3}{3*10^4} + \frac{60*10^3*(50)^2}{225*10^6}$$

$F_c = 2 + 0.66$

$\boxed{F_c = 2.66\ N/mm^2}$

a) Simentaneously tension — No losses.

b) Successive tension.

Cable-1 : Cable 1 is anchorage & tensioned & anchorage → No losses due to the elastic deformation.

cable-2 : Cable 2 is tensioned & anchorage → losses due to Cable-1.

Cable 3: Cable 3 is tensioned & anchoraged → loss due to cable 1 + cable 2.

Cable 2 Loss:

Loss of Stress in Cable-1 $= \alpha_e f_c$

$= 6(2.66)$

$= 15.96\ N/mm^2$

Cable 3 Loss:

Loss of stress in cable 1 $= \alpha_e * f_c = 6*2.66$

$= 15.96\ N/mm^2$

Loss of stress in cable 2 $= \alpha_e * f_c = 6*2.66$

$= 15.96\ N/mm^2$

The total loss of stress due to elastic deformation of concrete in Cable 1 =

$= 15.96 + 15.96$

$= 31.92\ N/mm^2$

Cable 2 $= 15.92\ N/mm^2$

Cable 3 $= 0$

Average loss of stress considering all the 3 cable

$$\frac{31.92+15.92+0}{3} = 15.94\ N/mm^2$$

It can be show that if the number of wires strands, bars are large. The loss due to elastic shortening does not exceed one half of the corresponding loss with pre-tensioning.

Avg. stress $\neq \frac{1}{2}(\alpha_e)(f_c)(n)$ n = no. of cable

$= \frac{1}{2}(6)(2.66)(3)$

$= 23.94\ N/mm^2$

$15.94 \neq 23.94\ N/mm^2$

Shrinkage :

3) A concrete beam is prestressed by a cable carrying an intial prestressing force of 300 kN. The c/s area of wires in the cable is $300\,mm^2$ calculate the percentage loss of stress due to shrinkage of concrete using IS: 1343- recomdations assuming the beam to be a) Pretensioned b) post tensioned

Assume ; $E_s = 210\,kN/mm^2$ & Age of concrete at transfer = 8 days.

Sol : Given data ;

$P = 300\,kN$

c/s area $= 300\,mm^2$

$t = 8$ days

a) Pre tensioned $\varepsilon_{cs} = 300 * 10^{-6}$

Loss of stress $= \varepsilon_{sc} * E_s$

$= 300 * 10^{-6} * 210 * 10^{+3}$

$= 63\,N/mm^2$

$= 0.063\,kN/mm^2$.

b) post tensioned $= \dfrac{200 * 10^{-6}}{\log_{10}(T+2)}$

$= \dfrac{200 * 10^{-6}}{\log_{10}(8+2)}$

$= 200 * 10^{-6}$

Loss of stress $= \varepsilon_{cs} * E_s$

$= 200 * 10^{-6} * 210$

$= 0.042\,N/mm^2$.

Intial stress $= \dfrac{300 * 10^3}{300}$

$= 1\,kN/mm^2$

Precentage loss for Pretensioned $= \frac{\;}{1} * 100$

$= 6.3\,\%$.

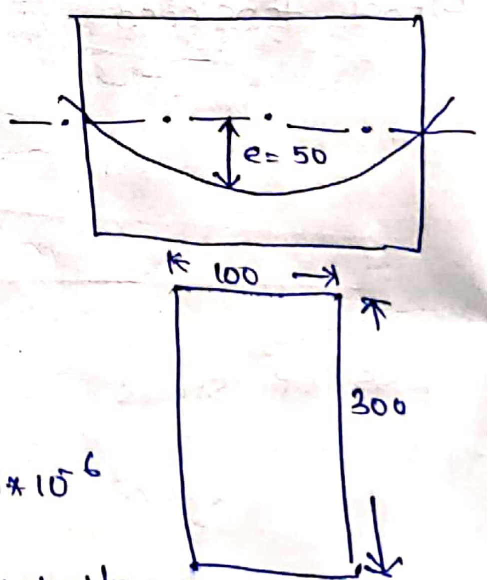
Percentage loss for post-tensioned $= \frac{0.042}{1} * 100$

$= 4.2\,\%$.

Creep:

4) A post tensioned concrete beam rectangular section 100mm wide & 300mm deep is stressed by parabolic cables with 'zero' eccentricity at supports and an eccentricity of 50mm at the centre span. The area of cable is 200mm^2 and the intial stress of the cable is 1200 N/mm^2. If the ultimate creep strain is $30*10^{-6}$ mm/mm per N/mm^2 of stress and modolus of elasicity of steel is 210kN/mm^2. Calculate the loss of stress in steel only due to creep of concrete.

Sol: Given data.



B = 300mm

d = 100mm.

area = 200mm^2

e = 50mm.

stress = 1200 N/mm^2

creep strain $\epsilon_{cc} = 30*10^{-6}$

E_s = 210kN/mm^2

P = stress * Area

= 1200 * 200

= 240 kN $\Rightarrow$ $\boxed{P = 240*10^3\,N}$

Stress at level of steel $(F_c) = \frac{P}{A} + \frac{Pe^2}{I}$.

$I = \frac{bd^3}{12} = \frac{100(300)^3}{12}$

$\boxed{I = 225*10^6 \text{ mm}^4}$

Parabolic $= \frac{2}{3}$

$A = 100*300 = 30*10^3 \text{ mm}^2$

$F_c = \frac{240*10^3}{30*10^3} + \frac{2}{3}\left[\frac{240*10^3(50)^2}{225*10^6}\right]$

$= 8 + 2.66 \quad \simeq 9.77$

$\boxed{F_c = 10.66 \text{ N/mm}^2}$

ultimate creep strain $= \varepsilon_{cc} * F_c * E_s$

$= 30*10^{-6} * 10.66 * 210*10^3$

$= 67.15 \text{ N/mm}^2$

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Friction:

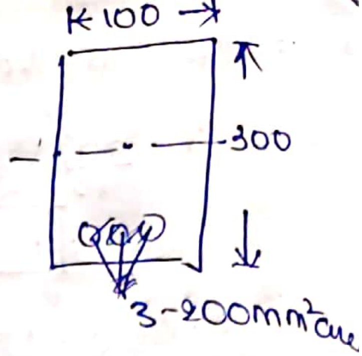
5). A concrete beam of 10m span, 100mm wide and 300mm deep is prestressed by 3cables. The area of each cable is 200mm² and the intial stress in the cable is 1200N/mm². Cable-1 is parabolic with an eccentricity of 50mm at the supports above the centroid. and the eccentricity is 50mm below the centre of span.

Cable-2 is also parabolic with zero eccentricity at supports and 50mm below the centroid.

Cable-3 is a straight with a uniform eccentricity 50mm below the centroid. If the cables are tensioned from one end only. Estimate the percentage loss of stress due to friction. Assume $\mu = 0.35$ & $K = 0.0015$/m. and eqn of parabola is written by

$y = \frac{4e}{L^2} * (L - x)$

Sol: Given data;

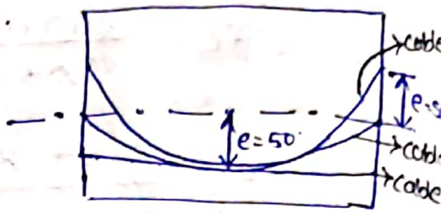


span (L) = 10m.

B = 100mm

D = 300mm

3 cables of 200mm² area (As).



Initial Stress = 1200 N/mm²

$\mu = 0.35$

$K = 0.0015/m.$

$y = \frac{4e}{L^2} x [L - x]$

$\frac{dy}{dx} = \frac{4e}{L^2} (x) [L - x]$

diff. the eqn.

$\frac{dy}{dx} = \frac{d}{dx}\left[\frac{4e}{L^2} (x)(L-x)\right]$

$= \frac{d}{dx}\left[\frac{4e}{L^2} (Lx - x^2)\right]$

at $x = 0$; $\frac{dy}{dx} = \frac{4e}{L^2}(L - 2(0))$

$= \frac{4e}{L}$.

Cable 1:

$\frac{dy}{dx} = \frac{4e}{L}$

$e_1 = 100mm$; $L = 10m = 10*10^3 m$

$\frac{dy}{dx} = \frac{4(100)}{10(10^3)} = 0.04 rad$

α = total slope = $2 \times (0.04) = 0.08 rad$

Cable 2:

$\frac{dy}{dx} = \frac{4e}{L}$

$e_2 = 50mm$; $L = 10m$

$\frac{dy}{dx} = \frac{4(50)}{10(10^3)} = 0.02 \text{ rad}$

α = total slope = 2(0.02) = 0.04 rad

cable 3:

$\frac{dy}{dx} = \frac{4e}{L}$

e = 0mm; L = 10m

$\frac{dy}{dx} = 0$; $\alpha = 0$

P_0 = intial stress * Area of cable

= 1200 (200) = 2400 kN

$\boxed{P = 240 \text{ kN}}$

Cable 1:

$P_x = P_0 [\mu\alpha + kx]$ (x = 10)

$= P_0 [0.35 * 0.8 + 0.0015 * 10] \Rightarrow P_0 [0.043]$

Cable 2, Loss of Stress.

$P_x = P_0 (\mu\alpha + kx)$

$= P_0 [0.35 * 0.04 + 0.0015 * 10]$

$= P_0 [0.029]$

cable 3: $P_x = P_0 (\mu\alpha + kx)$

$= P_0 [0.35 * 0 + 0.0015 * 10]$

$= P_0 [0.015]$

Cable NO	loss of stres	% loss of stress.
1	1200 * 0.043 = 51.6	$\frac{51.6}{1200} * 100 = 4.3\%$.
2	1200 * 0.029 = 34.8	$\frac{34.8}{1200} * 100 = 2.9\%$.
3.	1200 * 0.015 = 18	$\frac{18}{1200} * 100 = 1.5\%$.

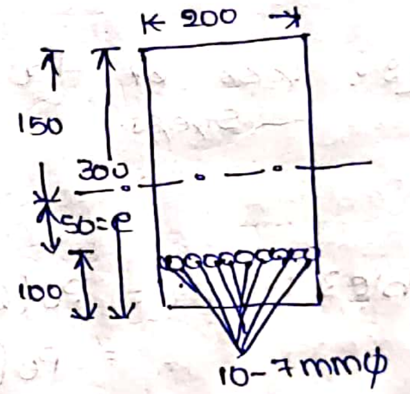
Total losses:

6) A Pretensioning beam 200mm wide and 300mm deep is Prestressed by 10wires of 7mm diameter initially Stressed to 1200N/mm² with their centroids located 100mm from the soffit. Find the maximum stresses in concrete immediately after transfer, Allowing only for elastic shortening (deformation)

If the concrete undergoes further shortening due to creep & shrinkage while their is a relaxation of 5% of steel stress. Estimate the final percentage of loss of stress in the wires using IS:1343 recomendations and the following data. $E_s = 210\ kN/mm^2$, $E_c = 5700\sqrt{F_{cu}}$, $f_{cu} = 42\ N/mm^2$; creep coefficient (ϕ) = 1.6 the total residual shrinkage strain = 3×10^{-4} (ε_{cs}).

7-1-2020

Sol: Given data,



$B = 200mm$

$D = 300mm.$

No. of bars = 10 - 7mm ϕ.

$e = 50mm$

Stress = $1200N/mm^2$

$f = \frac{P}{A}.$

$P = F*A = 1200 * 10 \cdot \frac{\pi}{4}(7)^2$ $\quad n \cdot \frac{\pi}{4} d^2$ $= 384.84$

$P = 461.81 \times 10^3 N$

$\boxed{P = 461.81 kN}$

$f_c = 42 N/mm^2$

$E_c = 5700\sqrt{F_{cu}}$

$= 5700\sqrt{42}$

$\boxed{E_c = 36.94\ kN/mm^2}$

$\varepsilon_{cs} = 3 \times 10^{-4}$ (Pre tensioning)

$f_c = \frac{P}{A} + \frac{Pe^2}{I}$

$I = \frac{bd^3}{12} = \frac{(200)(300)^3}{12} = 450 \times 10^6 \text{ mm}^4$

$f_c = \frac{461.81 \times 10^3}{60 \times 10^3} + \frac{461.81 \times 10^3 (50)^2}{450 \times 10^6}$

$= 7.69 + 2.56$

$\boxed{f_c = 10.25 \text{ N/mm}^2}$

Elastic deformation $= \alpha_e f_c$

$= \left(\frac{E_s}{E_c}\right) f_c$

$= \left(\frac{210}{36.44}\right) 10.25 \times 10^{-3}$

$= 58.27 \text{ N/mm}^2$

force in wires immediately after trafer

Force = Stress * Area A_s (wires)

$= [1200 - 58.27] [384.84]$

$\boxed{P = 439.38 \text{ KN}}$

$f_c = \frac{P}{A} + \frac{Pe^2}{I}$

$= \frac{439.38 \times 10^3}{60 \times 10^3} + \frac{439.38 \times 10^3 \times (50)^2}{450 \times 10^6}$

$= 7.32 + 2.441$

$\boxed{f_c = 9.76 \text{ N/mm}^2}$

Total losses :

i) Elastic deformation :-

$\alpha_e f_c$

$\left(\frac{E_s}{E_c}\right) \times f_c$

$$= \left(\frac{210}{36.94}\right) \times 10.25 \times 10^{-3}$$

$$= 58.27 \text{ N/mm}^2$$

Creep:

$$\varepsilon_c = \phi \, \alpha_e f_c$$

$$= 1.6 \left(\frac{210}{36.94}\right) (9.76) \times 10^{3}) .$$ 5.68.

$$= 88.77 \text{ N/mm}^2$$

Shrinkage:

$$\varepsilon_s = \varepsilon_{cs} \times E_s$$

$$= 3 \times 10^{-4} \times 210 \times 10^{3}$$

$$= 63 \text{ N/mm}^2$$

Relaxation:

Relaxation of 5% steel $= \frac{5}{100}(1200)$

$= 60 \text{ N/mm}^2$

Total losses in steel $= 58.27 + 88.77 + 63 + 60$

$= 270.04 \text{ N/mm}^2$ ✓

Remaining stresses in wires $=$ Intial stress $-$ final stress

$= 1200 - 270.04$

$= 929.96 \text{ N/mm}^2$

% loss of stress $= \frac{270.04}{1200} \times 100$

$= 22.50\%$.

8-1-2020

Anchorage slip

7) A posttensioned cable of beam 10m long is initially tensioned to a stress of 1000 N/mm² at one end. If the tendons are curved so that the slope is 1 in 24 at each end with an area of 600 mm², calculate the loss of prestress due to friction given the following data:-

→ Coefficient of friction(μ) b/w duct & cable = 0.55

→ friction coefficient for wave effect(k) = 0.0015/m.

→ During anchoring if there is a slip of 3mm at the jacking end(Δ). Calculate the final force in the cable & percentage loss of prestress due to friction and slip. $E_s = 210$ kN/mm²

Sol:- Given data,

$$\text{length}(l) = 10\text{m}.$$

$$\text{Stress }(f) = 1000\text{N/mm}^2 \text{ at one end}.$$

$$\text{Slope} = 1 \text{ in } 24$$

$$\text{Area}(A) = 600\text{mm}^2.$$

$$\mu = 0.55$$

$$k = 0.0015/\text{m}$$

$$\Delta = 3\text{mm}$$

$$E_s = 210 \text{ kN/mm}^2.$$

$$P = F * A$$

$$= 1000 * 600$$

$$\boxed{P_0 = 600 \text{ kN}}.$$

Friction :-

$$P_x = P_0(\mu\alpha + kx).$$

(tendons > 1)

$$\alpha = 2\left(\frac{1}{24}\right)$$

$$\alpha = \frac{1}{12}$$

$$P_x = 1600 * 10^3 * \left[0.55\left(\frac{1}{12}\right) + 0.0015 \overset{l}{(10)}\right]$$

$\boxed{P_c = 60.83 kN/mm^2}$

Loss due to anchorage slip:

$$\Delta = \frac{PL}{A E_s}$$

$$P = \frac{\Delta A E_s}{L}$$

$$= \frac{3(600)(210 * 10^3)}{10 * 10^3}$$

$$\boxed{P = 37.8 kN}$$

Loss of force due to friction:

$f = P_c * A$

$= 60.83 * 600$

$\boxed{F = 36.94\ kN}$

Total loss of force due to friction & slip

$= 37.8 + 36.94$

$\boxed{f = 74.74 kN}$

$A - P_c$

Final force in cable $= 600 - 37.74$

$= 525.26\ kN$

% Loss of Prestress $= \frac{74.74}{600} * 100 = \frac{P}{A} *$

$= 12.45\ \%$

PRESTRESSING SYSTEMS

Types:

1) Mechanical devices .- Screw, Jacks.

2) Hydraullic devices.

Freyssnt, Gifford . 5-100 tonnes.

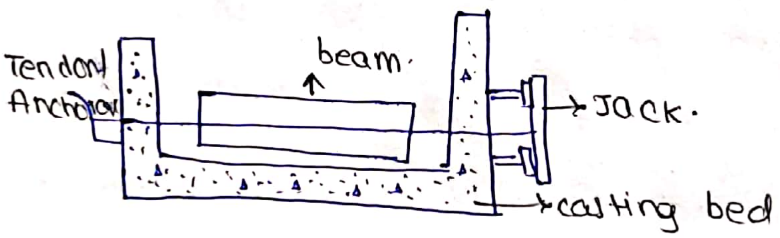
Bars - Leohardt - 200-600 tones.

3) Electrical devices -heat.

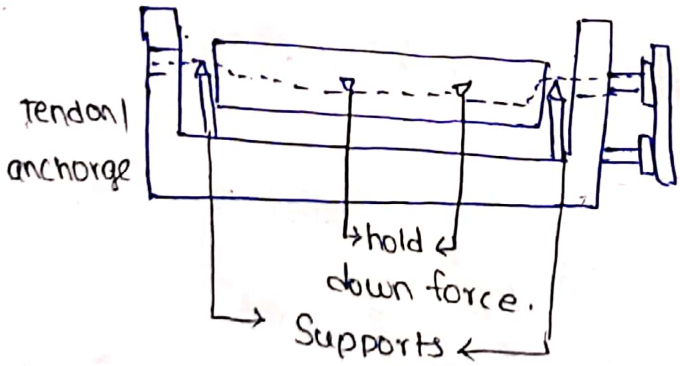
4) Chemical devices - cement.

Methods of Pre-stressing

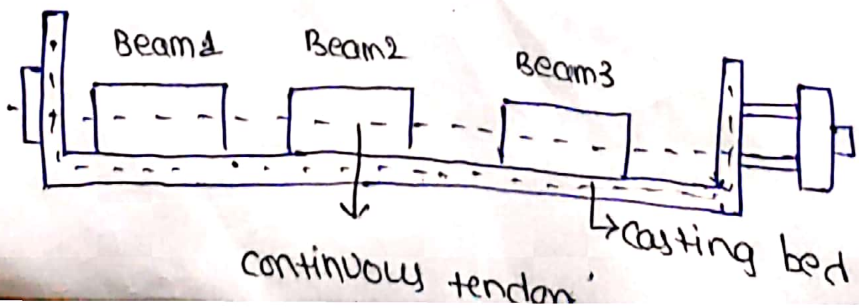
a) Beam with straight tendon.



b) Beam with variable tendon eccentrity



c) Hoyers long line system of pretensioning.



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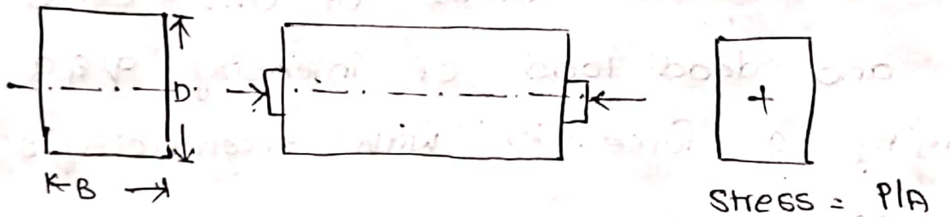
Assumptions of concrete:

* concrete is a homogeneous material. * within the range working stress both concrete and steel behave elastically.
* A plaine section before bending is assumed to remain plane after bending.

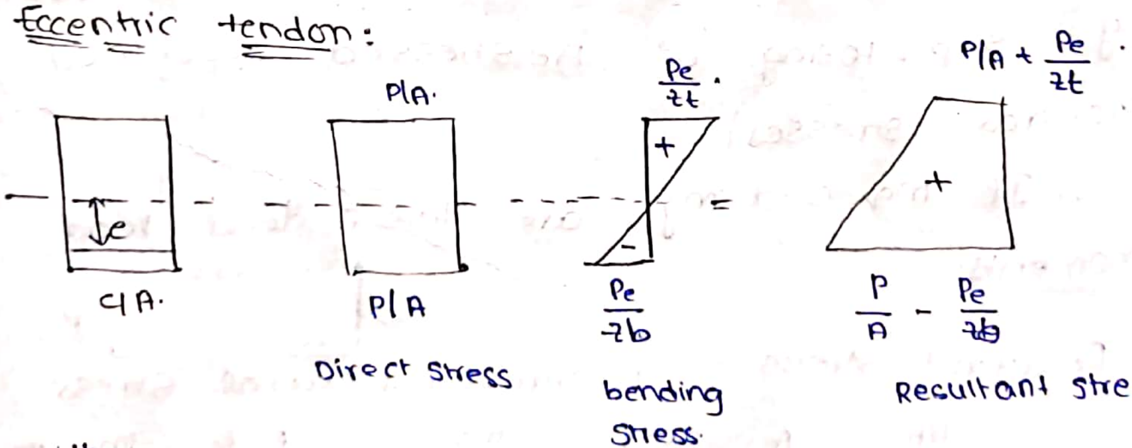
Analysis of Pre-stressed concrete:

Stress due to Pre stressing alone are generally combined stresses due to the action of direct load and bending resulting from an eccentrically applied load.

Concentric tendon:



Eccentric tendon:



Where:

I = moment of Inertia.

Zt = Section modolous at top of fiber

Zb = section modolous at bottom of fiber

P = Pre stressing force ;

e = Eccentricity of Pre stressing force.

M = moment.

A = area of c/s. section.

f_{top}, f_{bottom} = Pre stress in concrete developed at the top & bottom of fiber

+ → compression

− → tension.

y_{top}, y_{bottom} = Distance of the top & bottom fiber from the centroid of section.

i = Radius of gyration.

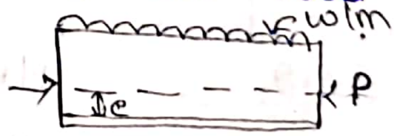
$z = \frac{I}{y}$.

<u>Resultant Stresses at a Section</u> :

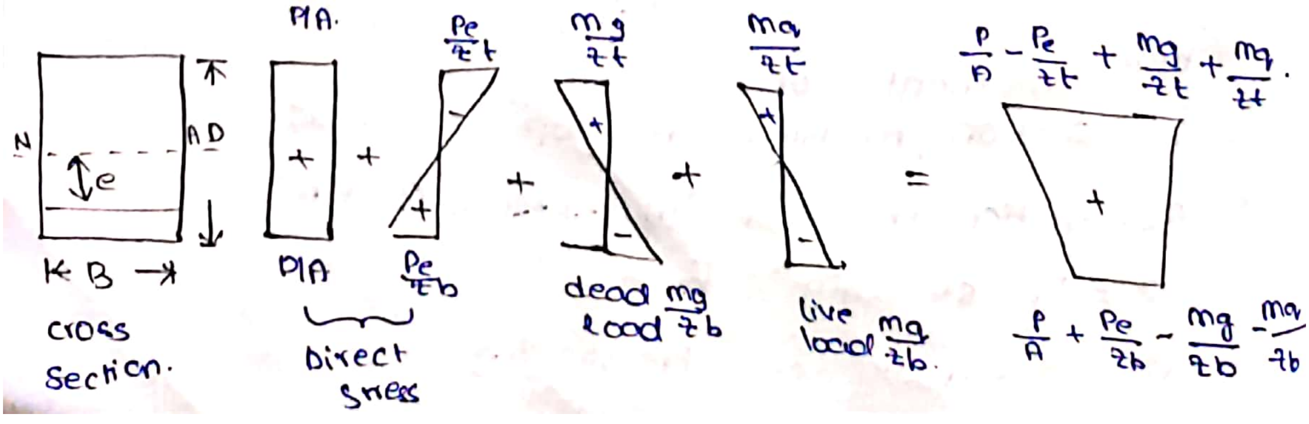
The concrete beam carries an UDL of a live load and dead load of intensity "q & g". and carrying a force 'P' with eccentricity 'e'.

Then the resultant stresses will be obtained by super posing of pre stressed & dead load (flexural stresses).

If m_q and m_g are live & dead load moments.



Resultant stress = Pre stress + flexural stress.



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The resultant stresses at the top and bottom fibers of concrete at any given section are obtained as f_{bottom}, f_{top}.

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z_t} + \frac{M_g}{Z_t} + \frac{M_q}{M_t}.$$

$$f_{bottom} = \frac{P}{A} + \frac{P}{Z_b} - \frac{M_g}{Z_b} - \frac{M_q}{Z_b}.$$

Problems:

1. A rectangular concrete beam 100mm wide and 250mm deep spanning over 8m is Prestressed by a straight cable carrying an effective Pre-stressing force of 250kN located at an eccentricity of 40mm. The beam supports a live load of 1.2kN/m.

a) Calculate the resultant stress distribution for the centre of span cross section of the beam assumming the density of concrete as 24kN/m3.

b) Find the magnitude of Prestressing force with an eccentricity of 40mm which can balance the Stresses due to dead load and live loads of the soffit of the centre span section (base)

Sol Given data,

c/s = 100mm × 250mm

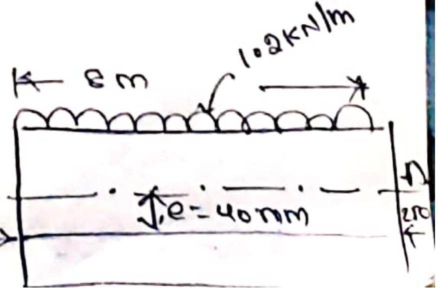
Span of the beam = 8m.

eccentricity (e) = 40 mm.

Prestressing force of beam = 250kN

live load = 1.2 kN/m.

Density of concrete = 24kN/m3



a) Stress due to direct load $= \frac{P}{A}$

$$= \frac{250*10^3}{100*250}$$

$$= 10\ N/mm^2$$

Bending resulting from eccentric load

$\left(\frac{Pe}{Zt} \text{ (or) } \frac{Pe}{Zb}\right)$

$$Z = \frac{I}{y} = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}.$$

Zt (or) Zb $\quad Z = \frac{100(250)^2}{6} = 1.04*10^6$

$$Pe = 250*10^3 * 40$$

$$Pe = 10*10^6\ N/mm^2$$

$$\frac{Pe}{Zb} = \frac{10*10^6}{1.04*10^6} = 9.6\ N/mm^2$$

Dead load and live load

Dead load - $mg = \frac{wl^2}{8}$.

$$\frac{wl^2}{8} = \frac{(0.1 * 0.25*24*10^3) * 8^2}{8}$$

$$= 4.61\ N/mm^2.$$

$$= 4.8*10^3\ N\text{-}m = 4.8*10^6\ N\text{-}mm$$

$$\frac{mg}{Z} = \frac{4.8*10^6}{1.04*10^6} = 4.61\ N/mm^2$$

live load $\quad mq = \frac{wl^2}{8}$

$$= \frac{1.2*10^3 * 8^2}{8}$$

$mq = 9.61*10^3\ N\text{-}m$ (or) $9.6*10^6 N\text{-}mm$

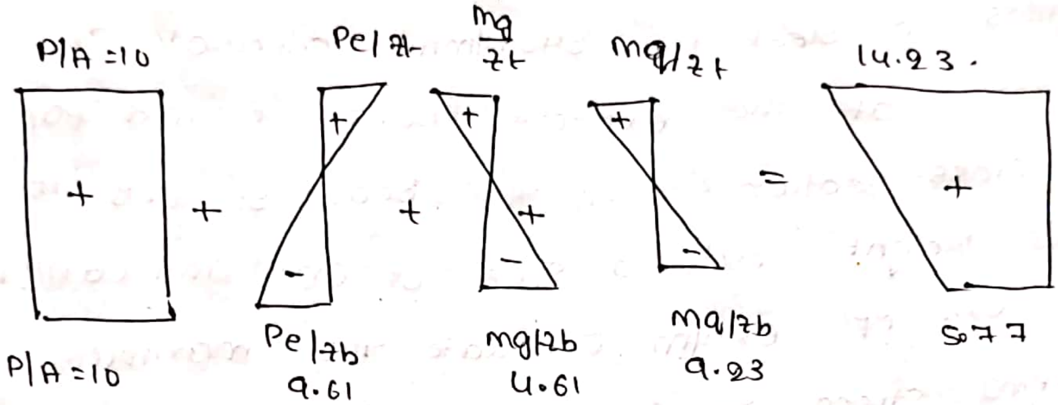
$$\frac{m_q}{z} = \frac{9.6\times10^6}{1.04\times10^6} = 9.93 \ N/mm^2$$

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z_t} + \frac{m_g}{Z_t} + \frac{m_q}{Z_t}$$

$$f_{top} = 10 - 9.61 + 4.61 + 9.93 = 14.93 \ N/mm^2.$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z_b} - \frac{m_g}{Z_b} - \frac{m_q}{Z_b}$$

$$= 10 + 9.61 - 4.61 - 9.93 = 5.77 \ N/mm^2.$$



b) If P is equal to the Prestressing force required to balance the stress at soffit than

$$\left[\frac{P}{A} \pm \frac{Pe}{Z}\right] = \left[\frac{m_g}{Z} \pm \frac{m_q}{Z}\right]$$

$$P\left[\frac{1}{A} \pm \frac{e}{Z}\right] = \left[\frac{m_g}{Z} \pm \frac{m_q}{Z}\right]$$

$$P\left[\frac{1}{100*250} + \frac{40}{1.04*10^6}\right] =$$

$$[4.61 + 9.23]$$

$$P[7.84*10^{-5}]$$

$$= 13.84$$

$$\therefore P = 176.5*10^3 \ N$$

$$\boxed{P = 176.5 \ kN}$$

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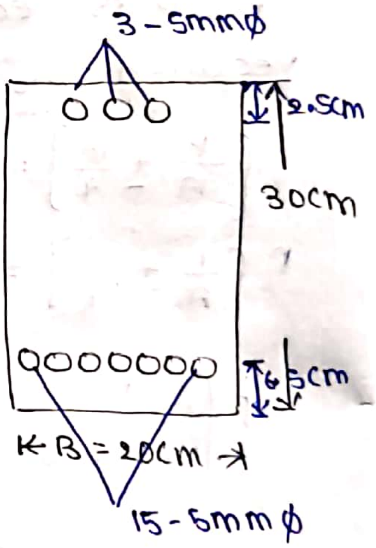
2. A rectangular concrete beam of cross section 30cm deep and 20cm wide is Pre-stressed by means of 15 wires of 5ϕmm diameter are located at 6.5cm from bottom of the beam and 3 wires of 5mm diameter are located at 2.5cm from top. Assuming the Pre-Stressing Stress in Steel is 840 N/mm². calculate the stresses at the extreme fibers of mid-span of cross-section. when the beam supports its own weight over a span of 6m. and carrying an UDL of 6kN/m. Evaluate the maximum working stress in concrete. The density of concrete is 24 kN/m3.



Sol: Given data:

c/s = 30cm * 20cm

Deep = 30cm = 300mm

wide = 20cm = 200mm.

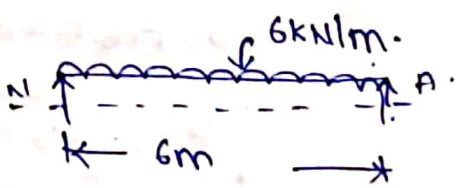
Stress in Steel = 840 N/mm²

15 bars of 5mm ϕ from 6.5cm top

3 wires of 5mmϕ at 2.5cm from bottom.

unit. wt. of concrete (density) = 24kN/m3.

UDL = 6kN/m.



Span length = 6m.

Distance of centroid in Prestressing force from the base.

from base (300-25 = 275)

$$y = \frac{(15*65) + (3*275)}{15+3}$$

$$\boxed{y = 100mm}$$

$$e = \frac{d}{2} - y$$

$$= \frac{300}{2} - 100$$

$$\boxed{e = 50mm}$$

$$\frac{P}{A} = 840\ N/mm^2.$$

c/s area

$$P = 840(\text{c/s area})$$

$$n \cdot \frac{\pi}{4}(d)^2$$

$$= 840\left(18*\frac{\pi}{4}(5)^2\right)$$

$$18\ \frac{\pi}{4}(5)^2$$

$$= 353.42$$

$$\boxed{P = 296.88\ kN}$$

Area of cross-section $= 200*300$

$$\boxed{A = 60,000\ mm^2.}$$

Section modolous $(z) = \frac{I}{y} = \frac{\frac{bd^3}{12}}{y/2}$

$$= \frac{bd^2}{6}.$$

$$z = \frac{(200)(300)^2}{6}$$

$$\boxed{z = 3*10^6\ mm^3}$$

Self weight => Mg $= (0.2*0.3*24)$ KN/m3

$$w_{mg} = 1.44\ KN/m$$

$$mg\ (\text{dead load}) = \frac{wl^2}{8} = \frac{1.44(6)^2}{8}$$

$$= 6.48\ KN\text{-}m$$

$$\boxed{mg = 6.48*10^6\ N\text{-}mm}$$

live load $(m_q) = \frac{wl^2}{8}$

$= \frac{6(6)^2}{8}$

$m_q = 27$ kN-m

$$\boxed{m_q = 27 * 10^6 \text{ N-mm}}$$

Due to direct load $= \frac{P}{A} = \frac{296.88 * 10^3}{6 * 10^4}$

$= 4.94 \text{ N/mm}^2$

$$\boxed{\frac{P}{A} = 5 \text{ N/mm}^2}$$

Due to eccentric load $= \frac{Pe}{Z} = \frac{296.88 * 10^3 * 50}{3 * 10^6}$

$= 4.94 \text{ N/mm}^2$

$$\boxed{\frac{Pe}{Z} = 5 \text{ N/mm}^2}$$

Resultant stress for dead load $= \frac{m_g}{Z}$

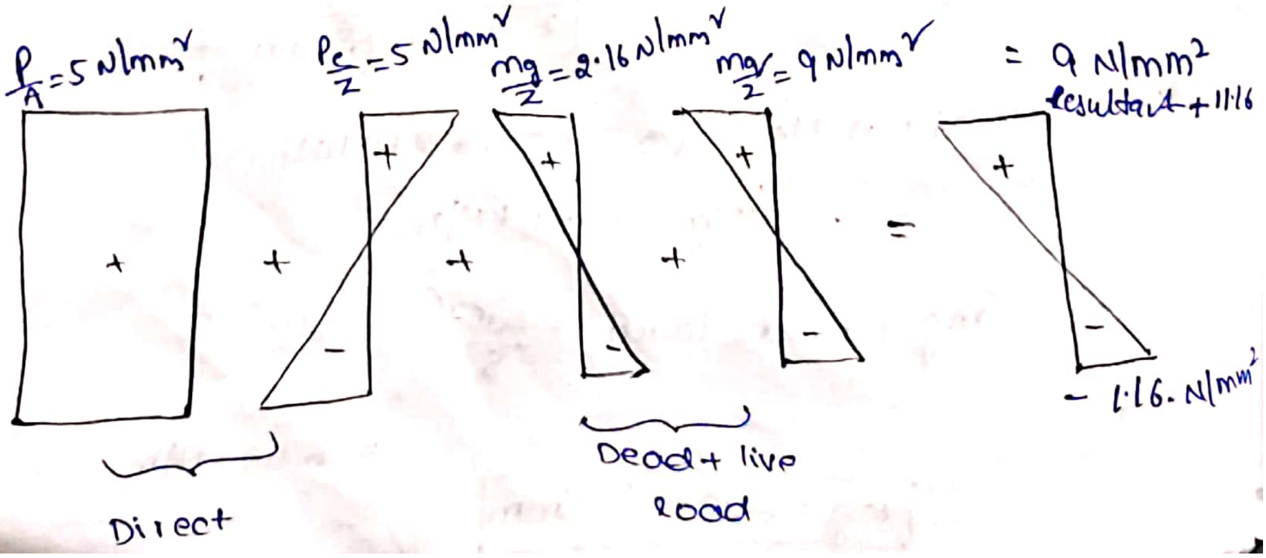
$= \frac{6.48 * 10^6}{3 * 10^6}$

$= 2.16 \text{ N/mm}^2$

Resultant stress for live load $= \frac{m_q}{Z}$

$= \frac{27 * 10^6}{3 * 10^6}$

$= 9 \text{ N/mm}^2$



$$f_{top} = \frac{P}{A} - \frac{Pe}{Z_t} + \frac{m_g}{Z_t} + \frac{m_q}{Z_t} = 5-5+2.16+9 = 11.16\ N/mm^2$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z_b} - \frac{m_g}{Z_b} - \frac{m_q}{Z_b} = 5+5-2.16-9 = -1.16\ N/mm^2$$

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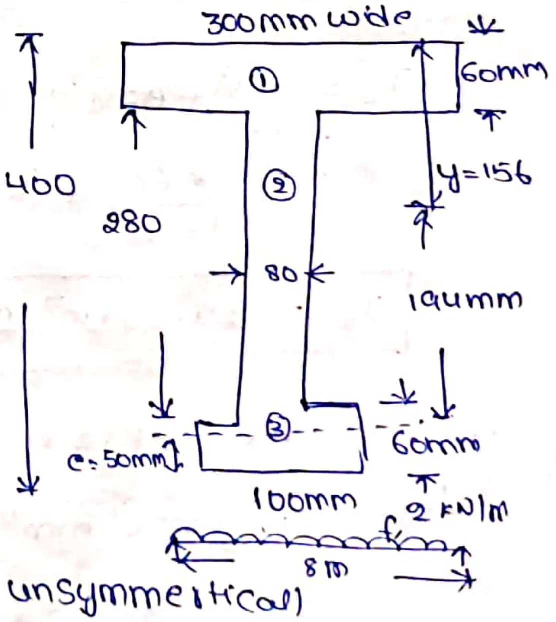
3) An unsymmetrical I-section beam used to support an imposed load of 25 kN/m over a span of 8m. The sectional details of top flange 300mm wide and 60mm thick and bottom flange 100mm wide and 60mm thick. Thickness of web is 80mm. Overall depth of the beam is 400mm at the centre of span. The pre-stressing force of 100 kN is located at 50mm from the soffit (base) of the beam. Estimate the stresses at the centre of span section of the beam for the following load conditions.

(a) Prestress + Self weight.

(b) Prestress + self weight + live load.

Sol



Area = $A_1 + A_2 + A_3$

$= (300 * 60) + (280 * 80) + (100 * 60)$

$= 46,400\ mm^2$

$\boxed{A = 0.0464\ m^2}$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} \quad \text{(for unsymmetrical)}$$

$$= \frac{\left[(300 * 60)\frac{60}{2} + \left(80 * 280 * \left(60 + \frac{280}{2}\right)\right) + (100 * 60)\left(60 + \frac{280}{2} + 60\right)\right]}{46400}$$

$$\therefore \bar{y} = \frac{540*10^3 + 4.48*10^6 + 2.22*10^6}{46400}$$

$$\boxed{\bar{y} = 156mm}$$

for symmetric ao $y = d/2$

Section modolus $Z = \frac{I}{y}$.

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{bd^3}{12} + A_1\bar{y}^2$$

$$= \frac{300*(60)^3}{12} + (300*60)(156 - \frac{60}{2})^2$$

$y = (\bar{y} - y_1)$

$$\boxed{I_1 = 29.11*10^7 \ mm^4}$$

$$I_2 = \frac{bd^3}{12} + A_2\bar{y}^2$$

$$= \frac{80*(280)^3}{12} + [80*280[156-60-\frac{280}{2}]^2]$$

$$\boxed{I_2 = 18.97*10^7 \ mm^4}$$

$$I_3 = \frac{bd^3}{12} + A_3\bar{y}^2$$

$$= \frac{100*60^3}{12} + (100*60)[400-156-\frac{60}{2}]^2]$$

$$\boxed{I_3 = 27.65*10^7 \ mm^4}$$

$$\therefore I = 29.11*10^7 + 18.97*10^7 + 27.65*10^7$$

$$\boxed{I = 75.73*10^7 \ mm^4}$$

$$Z_t = \frac{I}{y_t} = \frac{75.73*10^7}{156} = 4.8*10^6 \ mm^3$$

$$Z_b = \frac{I}{y_b} = \frac{75.73*10^7}{400-156} = 3.1*10^6 \ mm^3.$$

Dead load $m_g = \frac{wl^2}{8}$

w = Area of Section * Density of concrete.

$w = 0.0464 * 24$

$w = 1.13 KN/m^3$

$m_g = \frac{1.13(8)^2}{8}$

$= 8.83 KN\text{-}m$

$\boxed{m_g = 8.86 * 10^6 N\text{-}mm}$

m_q = live load $= \frac{wl^2}{8}$

$= \frac{2 * 8^2}{8} = 16 KN\text{-}m$

$\boxed{m_q = 16 * 10^6 N\text{-}mm}$

a) $\frac{P}{A}, \frac{Pe}{Zt}, \frac{m_g}{Zt}$

$f_{top} = \frac{100 * 10^3}{46400} - \frac{100 * 10^3 * 50}{4.8 * 10^6} + \frac{8.86 * 10^6}{4.8 * 10^6} = 0.$

$f_{bottom} = \frac{100 * 10^3}{46400} + \frac{100 * 10^3 * 50}{3.1 * 10^6} - \frac{8.86 * 10^6}{3.1 * 10^6} = 5.5 N/mm^2$

b) $\frac{P}{A}, \frac{m_g}{Z}, \frac{m_q}{Z}$

$f_{top} = \frac{100 * 10^3}{46400} - \frac{8.86 * 10^6}{4.8 * 10^6} + \frac{16 * 10^6}{4.8 * 10^6} = 3.29 N/mm^2$

$f_{bottom} = \frac{100 * 10^3}{46400} + \frac{8.86 * 10^6}{3.1 * 10^6} - \frac{16 * 10^6}{3.1 * 10^6} = 0.37 N/mm^2$

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Pressrue line (or) Thrust line and Resis...

At

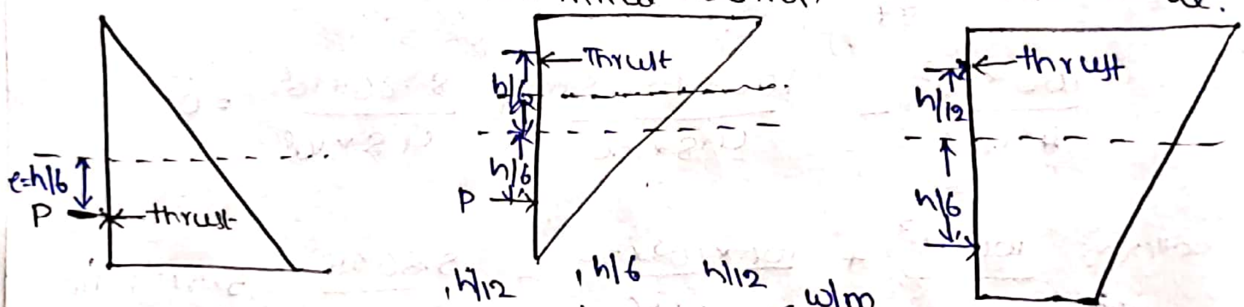
The locus of point of application of this force in any structure is treated as pressure line (or) Thrust line.

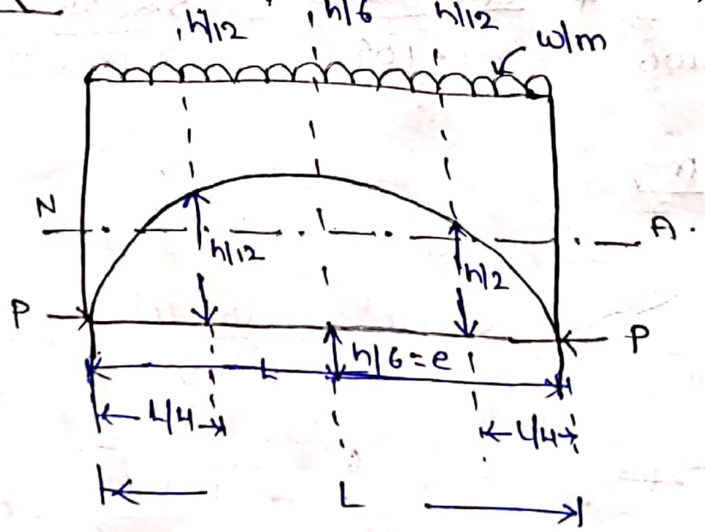
At any given section of a concrete beam the combined effect of Pre stressing force and externally applied load will result in distribution of concrete stresses. that can be resolved in a single force.

At support

At centre h/3 fed initial Position

at quater h/... from initial.





→ Location of Pressure line depends upon the magnitude & distribution upon on c/s section

→ magnitude & distribution of Stresses due to Pre-Stressing force.

Consider a Pre-stressing concrete beam shown in above fig. which is Pre-stressed by a force(P) and acting at eccentricity (e) and beam supports a U.D.L of intensity. w/m (or) q/m.

The load is such of magnitude that the bottom fiber stresses at the center span section is zero.

The above fig. shows that the resultant distribution of stresses at support, at center & at quater of spans.

<u>At support :</u>

No flexural stresses resulting from external loads. Therefore pressure line coincides with centeroid of the steel located at h/6.

<u>At center :</u>

Due to the external loading such that the resultant stresses developed is maximum at the top of the fiber and zero at the bottom of the fiber so that the pressure line shifted towards the top of the fiber by an amount of h/3 from its intial position.

<u>At quater of span (l/4)</u>

The external moment at l/4th of span is being smaller in magnitude, the pressure line shifts towards the corresponding the smaller at h/4 from its intial postion.

The pressure (or) thrust line concept can also be used for the calculated stresses. Generally these stresses can be calculated by using C-Line method (or) Internal Resisting couple method.

And concrete beam is analysed as a plain elastic beam.

P = external compressive force with a constant tensile force in tendons throughout the span.

At any section of a loaded Pre-Stressed beam, equilibrium maintained by satisfying the equations: $h = 0$ & $m = 0$

If m = bending moment at the section due to dead & live loads.

e = eccentricity of tendon

$t = P$ = Pre stressing force in tendon.

The moment equilibrium yields the relation

$M = Pa = Ca$ $\quad m = Pa = Ca$

$$\boxed{a = \frac{m}{P}}$$

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The shift of Pressure line e' from the centroidal axis is obtained by $e' = a - e$

Then the resultant stresses at the top and bottom fibers are

$$f_{top} = \frac{P}{A} + \frac{Pe'}{Z_t}$$

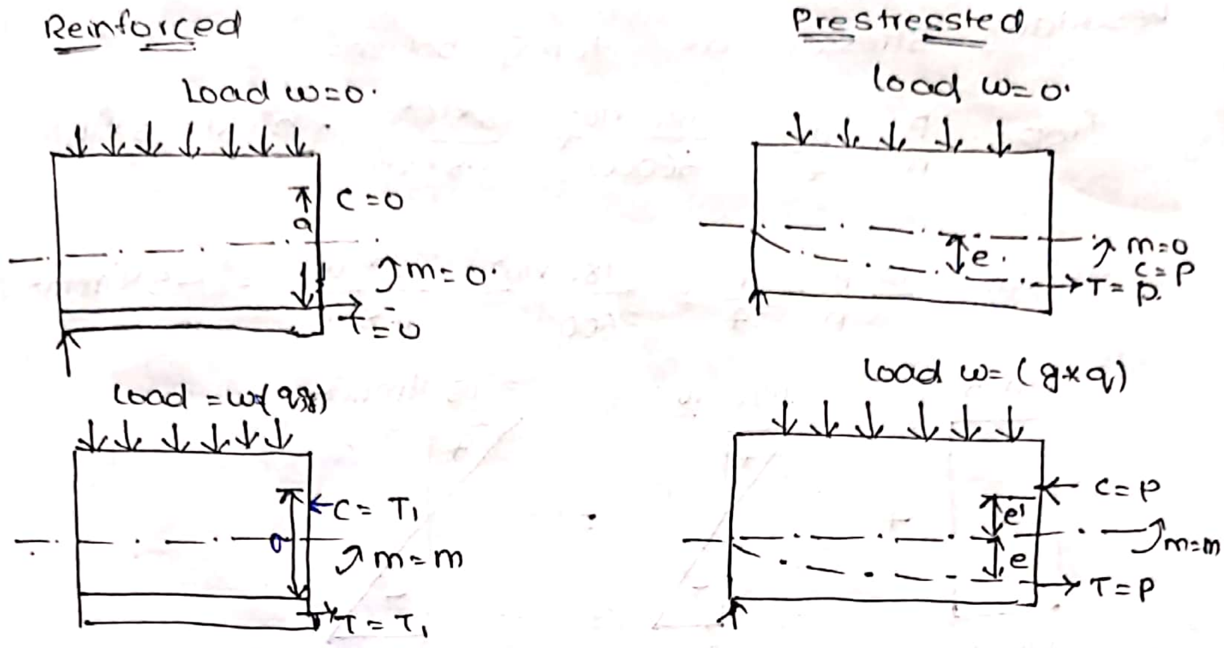
$$f_{bottom} = \frac{P}{A} - \frac{Pe'}{Z}.$$

$$e' = a - e$$

$$\Rightarrow a = \frac{m}{P}$$

$$e' = \left(\frac{m}{P}\right) - e.$$

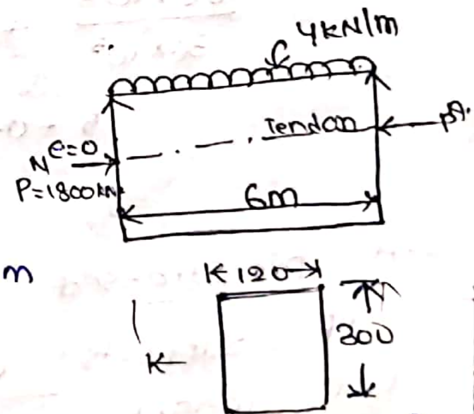
Load Carrying mechanism of Reinforced concrete & Prestressed concrete of beam section



1. A prestresed concrete beam with a rectangular c/s 120mm X 300mm deep. supports an U.D.L of 4 KN/m. per including its own weight. The effective span of beam is 6m. The beam is prestressed concentrically by a cable of carrying a force of 180 KN. Locate the pressure line in the beam.

20/12/19

Given data



wide $b = 120mm$

deep $d = 300mm$

Live & dead load = 4 KN/m

span = 6 m

force = 180 KN.

e = 0 [prestressed concentrically]

$$m = \frac{wl^2}{8} = \frac{(4)(6)^2}{8} = 18 \text{ KN-m} = 18\times10^6 \text{ N-mm}$$

$$Ar = \frac{m}{P} =$$

Area of c/s (A) = $120 \times 300 = 36000 mm^2$

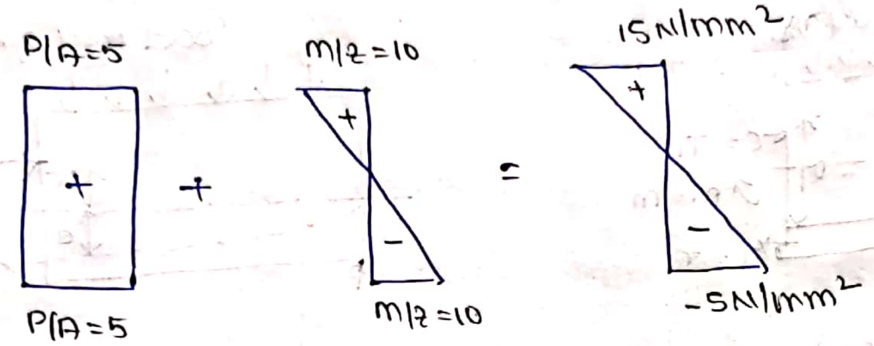
$$Z = \frac{bd^2}{6} = \frac{(300)^2(120)}{6} \qquad \left(\because \frac{Pe}{Z} = 0\right)$$

$$Z = 1.8 \times 10^6 \text{ mm}^3.$$

Resultant stresses at top & bottom.

$$f_{top} = \frac{P}{A} + \frac{M}{Z} = \frac{180 \times 10^3}{36000} + \frac{18 \times 10^6}{1.8 \times 10^6} = 5 + 10 = 15 \text{ N/mm}^2$$

$$f_{bottom} = \frac{P}{A} - \frac{M}{Z} = \frac{180 \times 10^3}{36000} - \frac{18 \times 10^6}{1.8 \times 10^6} = -5 \text{ N/mm}^2$$



If N = resultant thrust in the section

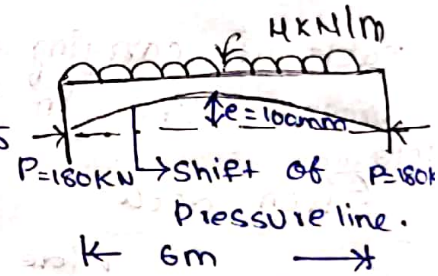
e = corresponding eccentricity. (shift of pressure.

$$\frac{N}{A} + \frac{Ne}{Z}.$$

$$\frac{180 \times 10^3}{36 \times 10^3} + \frac{180 \times 10^3 (e)}{1.8 \times 10^6} = 15$$

$$5 + 0.1(e) = 15$$

$$\boxed{e = 100\text{mm}}$$



2. A Pre-Stressed beam of section 120mm * 300mm deep supports a UDL of 4KN/m including its own weight. The effective span of beam is 6m. The beam is Prestressed eccentrically by a cable carrying a force 180KN and locate with an eccentricity of 50mm. Determine the location of Pressure line in the beam. and plot its position at 1/4 of the span & central span section.

Sol: Given data

i) Area = 120 * 300

$A = 36000\ mm^2$

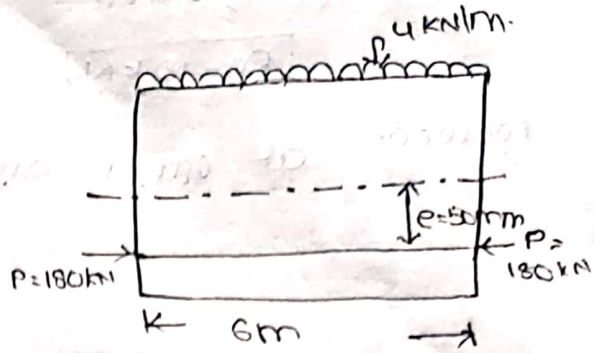
UDL = 4kN/m.

Span length (l) = 6m.

Force (P) = 180 kN

$= 180 * 10^3\ N$

eccentricity (e) = 50mm.



$$m = \frac{wl^2}{8} = \frac{4(6)^2}{8} = 18\ kN\text{-}m = 18 * 10^6\ N\text{-}mm$$

$$Z = \frac{bd^2}{6} = \frac{120(300)^2}{6} = 1.8 * 10^6\ mm^2$$

Resultant stresses at top & bottom.

$$f_{top} = \frac{P}{A} + \frac{Pe}{Z} + \frac{m}{Z} = \frac{180*10^3}{36000} - \frac{180*10^3*5}{1.8*10^6} + \frac{18*10^6}{1.8*10^6}$$

$= 5 - 5 + 10$

$= 10\ N/mm^2$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{m}{Z} = 5 + 5 - 10$$

$= 0\ N/mm^2$.

Shift of Pressure line (a) $= \frac{m}{P}$.

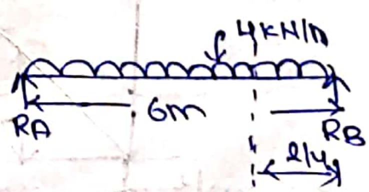
$$= \frac{18*10^6}{1.8*10^6} \Rightarrow \boxed{a = 100mm}$$

at centere.

ii) Resultant Stresses at quater of span (l/4)

$R_A + R_B = 6(4)$

$R_A + R_B = 24$



Taking moment about R_A.

$R_B * l - w(l)(l/2) = 0$

$R_B(6) - 4(6)(6/2) = 0$

$\boxed{R_B = 12\ kN}$

$R_A + R_B = 24\,kN$

$\boxed{R_A = 12\,kN}$

$\boxed{\therefore M = \frac{3wl^2}{32}}$

moment at $l/4$ distance $= R_B * l/4 - w * l/4 * \frac{l/4}{2}$

$= 12\left(\frac{6}{4}\right) - 4\left(\frac{6}{4}\right) * \frac{6/4}{2}$

$= 13.5\ kN\text{-}m$

$M = 13.5 \times 10^6\ N\text{-}mm$

$f_{top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z}$

$= \frac{180 * 10^3}{360 * 10^3} - \frac{180 * 10^3 * 5}{1.8 * 10^6} + \frac{13.5 * 10^6}{1.8 * 10^6}$

$= 5 - 5 + 7.5$

$f_{top} = 7.5\ N/mm^2$

$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$

$= 5 + 5 - 7.5$

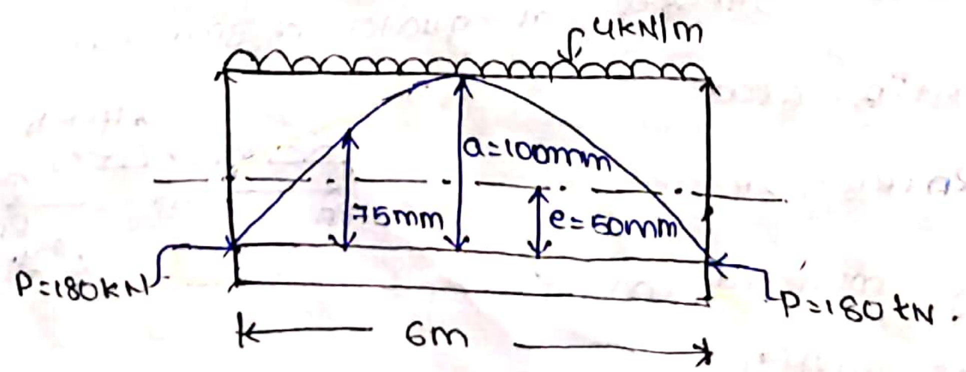
$= 10 - 7.5$

$= 2.5\ N/mm^2$

Shift of Pressure $(a) = \frac{M}{P}$

$= \frac{13.5 * 10^6}{180 * 10^3}$

$\boxed{a = 75\,mm}$

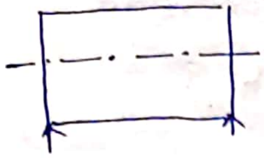
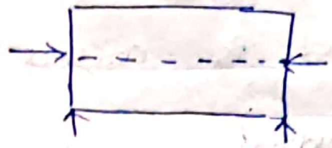


21/12/19

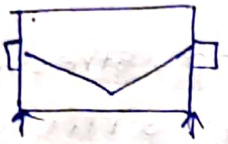
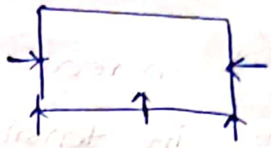
Cable profile

Reaction of tendon.

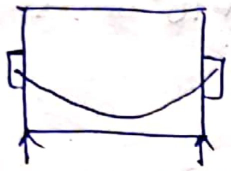
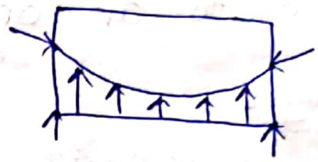
a) Straight Tendon.

b) Bent tendon.

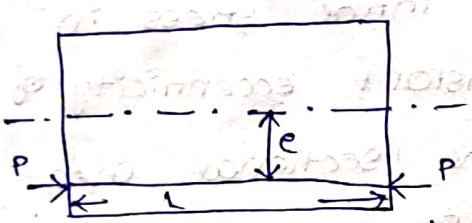



c) Curved Tendon

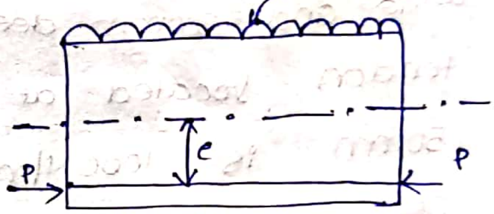



27/12/19

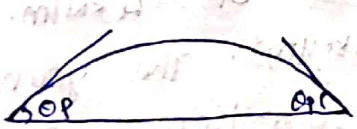
Stresses in Tendon.



effect of Prestressing force on tendon rotation of concrete beam.



Effect of transverse loads on rotation of concrete beam.



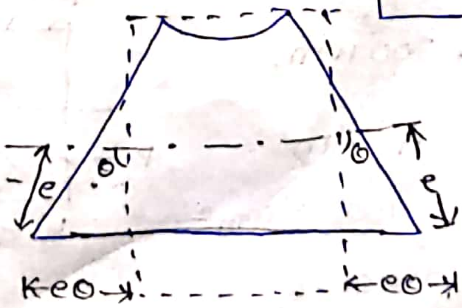
Hogging

Sagging

$$\theta_p = \frac{\overset{1/2\,PeL}{\text{Area of bending moment diagram}}}{\text{Flexural Rigidity (EI)}} \Rightarrow \boxed{\frac{PeL}{2EI} = \theta_p}$$

$$\theta_1 = \frac{\text{Area of B.M}}{\text{flexural rigidity (EI)}} = \frac{\frac{2}{3}\cdot\frac{wl^2}{8}\times l/2}{EI}$$

$$\boxed{\theta_1 = \frac{wl^3}{24EI}}$$



$\theta_1 > \theta_p$

$\theta = \theta_1 + \theta_p$.

Total elogation $= 2e\theta$; Strain $= \frac{2e\theta}{L}$

$$\boxed{\text{Stress} = E_s \times \frac{2e\theta}{L}}$$

Cracking moment:

Bending moment at which the visible cracks are developed in a Prestressing concrete member is called cracking moment

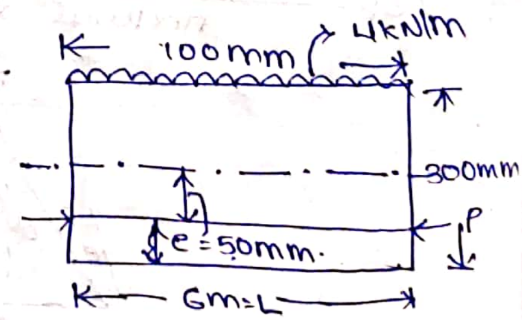
width = 0.01mm – 0.02mm.

Stresses increase of transverse load, stresses results in tensile stresses and at soffit of beam to an amount of $80-100 * 10^{-6}$ units.

Problems

1. The cross-section of a Pre-stressed concrete beam used over a span of 6m is 100mm wide and 300mm deep. The intial stress in tendon located at a constant eccentricity of 50mm is $1000 N/mm^2$. The sectional area of tendons is $100mm^2$. Find the percentage increase in stress in the wires when the beam supports a live load of 4 kN/m. The density of concrete is $24 kN/m^3$. The young's modulus of concrete $E_c = 36 kN/mm^2$. The youngs modulus of steel $E_s = 210 kN/mm^2$.

Sol: Given data.



C/S = 100 * 300 mm.

UDL (w) = 4 kN/m

Stress = $1000 N/mm^2$

$\frac{P}{A}$ = stress

P = Stress * Area = 1000 * 100 (C/S)

$\boxed{P = 100kN}$

$$I = \frac{bd^3}{12}$$

$$= \frac{100(300)^3}{12}$$

$$= 225 \times 10^6 \ mm^4$$

$$\theta_p = \frac{PeL}{2EI}$$

$$= \frac{(100 \times 10^3 \times 50 \times 6 \times 10^3)}{2 \times 36 \times 10^3 \times 225 \times 10^6} = 0.0018 \text{ rad}$$

$$\theta_1 = \frac{wl^3}{24EI}$$

Self wt = (Area * unit. of concrete)

$$w = 300 \times 100 \times 24 \times 10^3$$

$$= 720 \times 10^3 \ 1 \times 1. \quad = 4.72 \text{ KN/m}.$$

$= 4.72$ KN/m.

$= 4.72$ KN/1000mm

$= 0.0047$ KN/mm

0.0047 KN/mm

$$\theta_1 = \frac{(4.72)(6 \times 10^3)^3}{24 \times 36 \times 10^3 \times 225 \times 10^6}$$

$$\theta_1 = \frac{(0.00472)(6000)^3}{24 \times 36 \times 225 \times 10^6} = 5.22 \times 10^{-3}$$

= KN-mm.

$$\theta_1 = 0.0052 \text{ rad}$$

$$\theta_1 > \theta_p$$

$$\theta = \theta_1 - \theta_p$$

$$= 0.0052 - 0.0018$$

$$\boxed{\theta = 0.0034 \text{ rad}}$$

Stresses in tendon $= E_s \times \frac{2e\theta}{L}$

$$= 210 \times \frac{2(50)(0.0034)}{(6 \times 10^3)}$$

$$= 11.9 \ N/mm^2 \simeq 12 \ N/mm^2$$

The percentage increase in stresses $= \frac{12}{1000} \times 100$

$= 1.2\%$

30/12/19

Cracking moment :-

Q. A rectangular concrete beam of c/s section 120mm*300mm is prestressed by a straight cable carrying by an effective force of 120KN at an eccentricty of 50mm. The beam supposed imposed load of 3.14 KN/m over a span of 6m. The modolus of rupture of concrete is 5N/mm². Evaluate the load factor aganist the cracking. Assuming the density of concrete is 24 KN/m³

Sol : Given data;

c/s section = 120mm * 300mm.

force (P) = 120 KN.

eccentricity (e) = 50mm.

live load = 3.14 KN/m.

length (l) = 6m.

modolus of rupture of concrete = 5 N/mm²

density of concrete = 24 KN/m³.

Area (A) = $36 * 10^3$ mm²

$$Z = \frac{bd^2}{6} = \frac{(120)(300)^2}{6} = 1.8 * 10^6 \text{ mm}^3$$

$$= 18 \text{ m}^3$$

stresses at top & bottom

$$f_{top} = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z} \quad (M_q + M_g)$$

$$f_{bottom} = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

$$f_{top} = \frac{120*10^3}{36*10^3} - \frac{120*10^3*50}{1.8*10^6} + \frac{(120*300*24*10^3)+(3.14*10^3)}{1.8*10^6}$$

$= 5-5+480$

$= 480\ N/mm^2$

$$f_{bottom} = \frac{120*10^3}{36*10^3} + \frac{120*10^3*50}{1.8*10^6} - \frac{(120*300*24*10^3)+(3.14*10^3)}{1.8*10^6}$$

$= 5+5-480$

$= -470\ N/mm^2$] x end of this unit

Tensile stress at bottom fiber increases to $5N/mm^2$ tensile stress.

$= z * 5\ N/mm^2$

$= 1.86*10^6 * 5$

$= 9*10^6$ N-mm

$=$ 9KN-m.

Cracking moment = Intial + additial moment.

$= 18+9$

$= 27$

Load factor $= \frac{\text{cracking moment}}{\text{working moment}} = \frac{27}{18} = 1.5$

Load balancing ~~moment~~ concept:

1. A rectanguean concrete beam of 300mm wide & 800mm deep supports two concentrated loads of 20kN each at the third point of span 9m. (a) suggest a suitable cable profile. if the eccentricity of the cable profile is 100mm for the middle third portion of beam, calculate the pre-stressing force required to balance the bending effect of the

concentrated loads. (neglect the self weight of the beam).

(b) For the same cable profile find the effective force in the cable if the resultant stress due to self weight, imposed loads and the prestressing force is zero at the bottom fiber of the midspan section. Assume density of concrete 24 kN/m³.

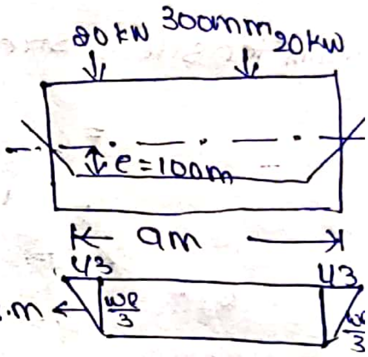
Sol : Given data

c/s of beam = 300*800 mm

force(P) = 20kN.

e = 100mm.

Area = $24 \times 10^4 \ mm^2$



$z = \frac{bd^2}{6} = \frac{(300)(800)^2}{6}$

$= 32 \times 10^6 \ mm^3$

a) Prestressing force (P) $= \frac{Wl}{3e} = \frac{20 \times 9}{3 \times 0.1} = 600$ kN.

b) $w_g = [0.3 \times 0.8 \times 24]$

$= 5.76$ kN/m.

$m_g = \frac{wl^2}{8} = \frac{5.76 \times 9^2}{8}$

$= 58.32 \times 10^6$ N-mm.

m_q = moment at centre due to load

$= \frac{2wl}{3} = \frac{wl}{3} = \frac{(5.76)9}{3}$

$= 60 \times 10^6$ N-mm

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$$\left(\frac{P}{A}+\frac{Pe}{z}\right) = \left(\frac{mg}{z}+\frac{mq}{z}\right)$$

$$P\left[\frac{1}{A}+\frac{e}{z}\right] = \frac{1}{z}\left[mg+mq\right]$$

$$P\left[\frac{1}{24\times10^4}+\frac{100}{32\times10^6}\right] = \frac{1}{32\times10^6}\left[58.32\times10^6+60\times10^6\right]$$

$$P\left[7.29\times10^{-6}\right] = 3.69\text{ kN.}$$

$$\boxed{P = 5.06\text{ kN}}$$

31-12-19

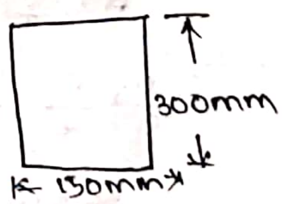
(2) A ~~Plan~~ Pre-Stressed beam 150mm × 300mm deep is used over an effective span of 10m. The cable with zero eccentricity at support and linearly varying to 50mm at centre, carries an effective Pre stressing force of 500kN. Find the magnitude of the concretrated load(Q) located at the centre of the span for the following conditions at the centre of span section.

(a) If the load contracts the bending effect of the prestressing force (neglect the self weight of the beam.

(b) If the Pressure line Passes through the upper kern of the section under the action of the external load, self weight, and prestress.

Sol

Given data,



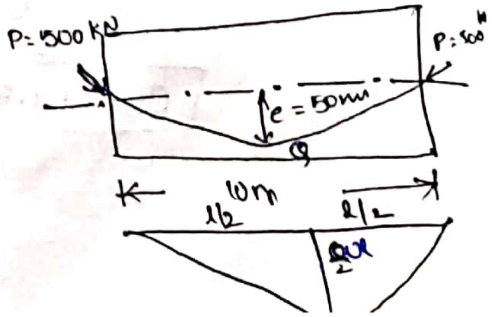
c/s area = 150*300

A = $45\times10^3 mm^2$

Span (l) = 10m.

eccentricity (e) = 50mm.

Force (P) = 500 kN



$$Z = \frac{bd^2}{6} = \frac{(150)(300)^2}{6}$$

$$= 2.25 * 10^6 \text{ mm}^3$$

(a)

$$M = Pe$$

$$M = \frac{wl}{2}$$

$$Pe = \frac{wl}{2}$$

$$2Pe/l = w$$

$$w = \frac{2(500)(50)}{(\frac{10}{2}) * 10^3}$$

$$Q \cdot \boxed{w = 10 \text{ KN}}$$

$$M = Mg + Mq$$

$$Mg = \frac{wl^2}{8} = \frac{10(10)^2}{8}$$

b)

$$Mg = \frac{wl^2}{8} = \frac{(0.15 * 0.3 * 24) * 10^2}{8} = 13.5 \text{ KN-m}$$

$$= 135 * 10^6 \text{ N-mm}$$

$$Mq = \frac{Ql}{4} = \frac{Q(10)}{4} = 2.5Q \text{ KN-m}$$

$$\frac{M}{Z} = \frac{Mg + Mq}{Z} = \frac{(13.5 + 2.5Q) * 10^6}{2.25 * 10^6}$$

$$\frac{P}{A} + \frac{Pe}{Z} = \frac{500 * 10^3}{45 * 10^3} + \frac{500 * 10^3 * 50}{2.25 * 10^6}$$

$$= 11.1 + 0.2/2 \ 11.11$$

$$\frac{P}{A} + \frac{Pe}{Z} = 22.22$$

$$\frac{P}{A} + \frac{Pe}{Z} = \frac{M}{Z}$$

$$22.22 = \frac{(13.5 + 2.5Q) * 10^6}{2.25 * 10^6}$$

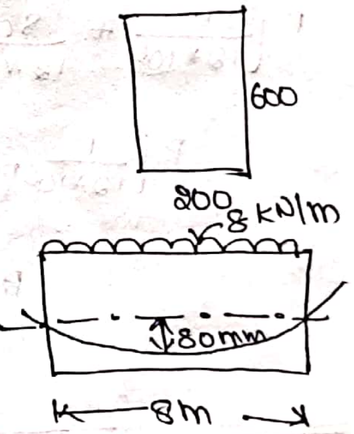
$$\boxed{Q = 14.59 \text{ KN}}$$

3) A Prestressed concrete beam of rectangular c/s section 200mm wide & 600mm deep. Supports a live load of 8 kN/m spanning over 8m. Find the effective Prestressing force in the Parabolic Cable having an eccentricity of 80mm at the centre of span and concentric at the supports for the following load conditions.

a) If the bending effect of the Pre-stressing force is nullified by the imposed load for the mid span section. (neglect the self weight of the beam).

b) If the resultant stresses due to self weight, live load & Prestressing force is zero at the soffit of the beam at centre of span section. Assume density (Dc) of concrete 24 kN/m³

Sol: Given data.



c/s = 200 * 600 mm

Area (A) = 12×10^4 mm²

live Load = 8 kN/m.

length(l) = 8 m.

eccentricity (e) = 80mm.

$$z = \frac{bd^2}{6} = \frac{(200)(600)^2}{6} = 12 \times 10^6 \text{ mm}^3.$$

a)

$$m = \frac{wl^2}{8}$$

$$Pe = \frac{wl^2}{8}$$

$$P = \frac{wl^2}{8(e)} = \frac{(8)(8)^2}{8(0.08)}$$

$$\boxed{P = 800 \text{ KN}}$$

(b) $\frac{P}{A}+\frac{Pe}{Z}-\frac{M}{Z}$.

$$\frac{P}{A}+\frac{Pe}{Z}=\frac{800*10^3}{12*10^4}+\frac{(800*10^3)(80)}{12*10^6}$$

$M = Mg + Mq$

$$Mq=\frac{wl^2}{8}=\frac{8(8)^2}{8}=64\,kN\text{-}m$$

$$Mg=\frac{(0.2*0.6*24)(8)^2}{8}=23.04\,kN\text{-}m$$

$$Mg+Mq=64+23.04=87.04$$

$$\frac{P}{A}+\frac{Pe}{Z}=\frac{Mg}{Z}+\frac{Mq}{Z}$$

$$P\left[\frac{1}{A}+\frac{e}{Z}\right]=\frac{1}{Z}(Mg+Mq)$$

$$P\left[\frac{1}{12*10^4}+\frac{80}{12*10^6}\right]=\frac{1}{12*10^6}\left[87.04*10^6\right]$$

$0.5*10^7$

$$P\left[\frac{1}{12*10^6}+\frac{80}{12*10^6}\right]=7.25$$

$$P[+4.96*10^6]=7.25$$

$$\boxed{P=48.46\,kN}$$

$$Mg=\frac{wl^2}{8}=\frac{0.12*0.300*24*16}{8}=3.88\,kN\text{-}m=3.88*10^6\,N\text{-}mm.$$

$$Mq=\frac{wl^2}{8}=\frac{3.14(6)^2}{8}=14.18\,kN\text{-}m\;;\;M=Mg+Mq=18\,kN\text{-}mm.$$

$$\frac{M}{Z}=\frac{18.6*10^6}{1.8*10^6}=10$$

$$f_{top}=\frac{P}{A}-\frac{Pe}{Z}+\frac{M}{Z}=5-5+10=10\,N/mm^2$$

$$f_{bottom}=\frac{P}{A}+\frac{Pe}{Z}-\frac{M}{Z}=5+5-10=0.$$

Cracking moment

Problem.

DESIGN FOR FLEXURAL RESISTANCE

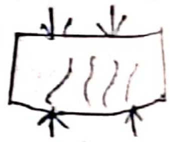
Types of flexural failure :-

It depends upon

1. % of reinforcement in the section.
2. Bond b/w concrete and steel/tension.
3. Compressive stregth of concrete.
4. Ultimate tensile strength of tendon.

Types :-

1. Fracture of steel in tendon
2. Failure of under reinforcement section.
3. Failure of over reinforced section.
4. Other modes of failure



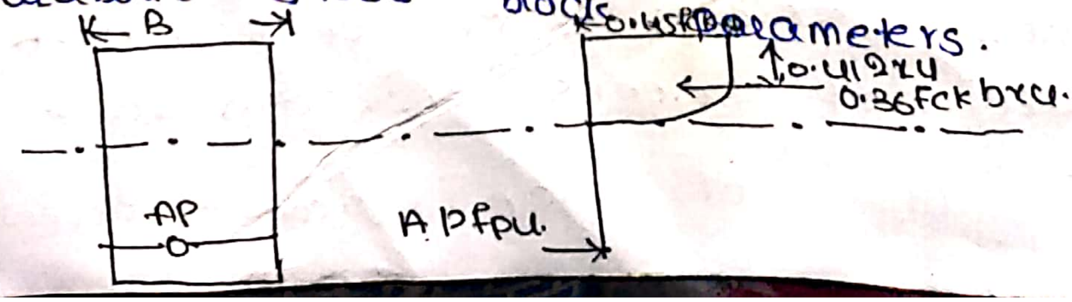
Pretensioned → Inadequate transmission length.
Post tensioned → Anchorage failure

IS code provisions :-

IS : 1343 - 1980 : Appendix-B
Pg: 59, 60.

Rectangular Section :-

The Indian Standard code method for computing the flexural strength of rectangular sections (or) T-sections in which the neutral axis lies with in the flange, is based on the rectangle and Parabolic stress blocks parameters.



$$M = f_{pu} A_p [d - 0.42 x_u].$$

where,

m = moment of resistance of the section.

f_{pu} = ultimate tensile stress in the tendons.

A_p = Area of Pre stressing tendon.

d = effective depth.

x_u = neutral axis depth.

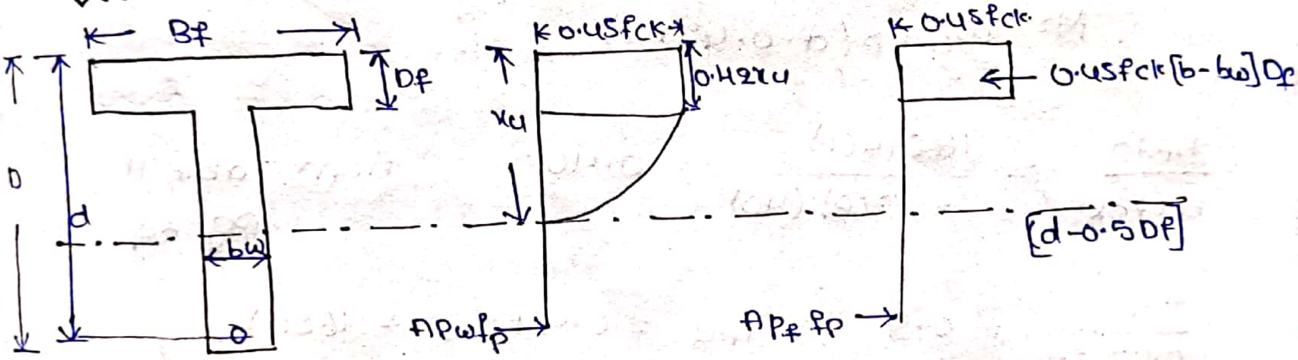
Based on the value of $\frac{A_p f_p}{bd f_{ck}}$ on.

The value of $\frac{f_{pu}}{f_p}$ & x_u are obtained from Table : 11 of IS : 1343-1980 NXB Pg : 59 & 60.

$f_{pu} \ngtr 0.45 f_b$.

1/2/2020

T-Section : [moment of resistance of flanged section]



where $x_u > D_f$

$$M_U = f_{pu} A_{pw} [d - 0.42 x_u] + 0.45 f_{ck} [b - b_w] D_f [d - 0.5 D_f]$$

$$A_{pf} = 0.45 f_{ck} [b - b_w] \left[\frac{D_f}{f_p}\right]$$

then $A_{pw} = A_p - A_{pf}$

effective reinforcement ratio $\frac{A_{pw} f_p}{b_d f_{ck}}$ (IS : 1343-1980) $b_d = b_w$.

A_{pw} = Area of Prestressing steel for web.

A_{pf} = Area of Prestressing steel for flange.

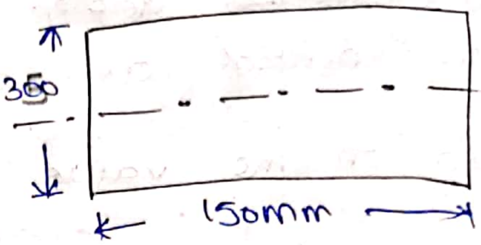
D_f = thickness of flange.

b_w = thickness of web.

The corresponding values of $\frac{f_{pu}}{0.87 f_b}$ & $\frac{x_u}{d}$ are obtained from table 11 pg : 59.

of 50mm. If fck = 40N/mm^2 and fp = 1600N/mm^2 and the area of prestressing steel (Ap) = 461 mm^2. Calculate the ultimate flexural strength of the section using IS code method.

Sol: Given data,



Wide (b) = 150mm

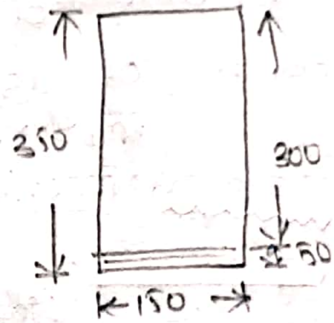
Deep (D) = 350mm

effective cover = 50mm

f_{ck} = 40 N/mm^2

fp = 1600 N/mm^2.

Ap = 461 mm^2



effective depth (d) = 350-50 = 300mm

$M = f_{pu} A_p (d - 0.42 x_u)$.

$$\frac{A_p f_p}{b d f_{ck}} = \frac{1600(461)}{150(300)(40)} = 0.40.$$

From Table 11
Pg: 59

$$\frac{f_{pu}}{0.87 f_p} = 0.9 \Rightarrow f_{pu} = 0.9(0.87 * 1600)$$

$f_{pu} = 1252.8$ N/mm^2

$$\frac{x_u}{d} = 0.783 \Rightarrow \cancel{x_u}\ 0.783(300) = 234.9\text{mm}.$$

$M = f_{pu} A_p (d - 0.42 x_u)$

$= 1252.8(461)[300 - (0.42 * 234.9)]$

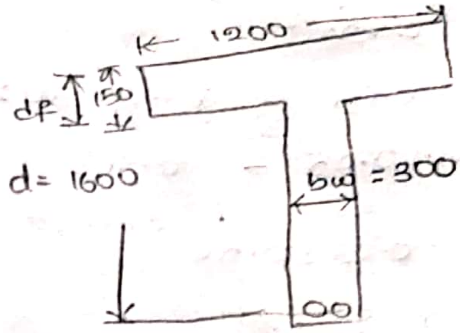
$= 577540 * [201.3]$

$\boxed{M = 116 * 10^{6} \text{ kN-mm}}$

1/2/2020.

2) A pretensioned T-section has a flange of 1200mm wide & 150mm thick. The width & depth of the rib 300mm and 1500mm resp. The high tensile steel as an area of 4700mm^2 and is located at an effective depth of 1600mm, If the characteristic strength of the cube concrete & tensile strength of steel is

40 N/mm² & 1600 N/mm² resp. Calculate the flexural
...ngth of T-section.



Given data,

$b_f = 1200mm$

$d_f = 150mm$

$b_w = 300mm$

$b d_0 = 150mm$

Area $(A_p) = 4700 mm^2$

effective depth (d) = 1600mm.

$f_{ck} = 40 N/mm^2$

Tensile strength $(f_p) = 1600 N/mm^2$.

Effective reinforcement ratio $= \frac{A_{pw} f_p}{b d f_{ck}}$

$= \frac{(4700)(300)(1600)}{}$

$A_{pw} = A_p - A_{pf}$

then $A_{pf} = 0.45 f_{ck} [b_f - b_w] \left[\frac{D_f}{f_p}\right]$

$= 0.45(40) [1200-300] \left[\frac{150}{1600}\right]$

$= 1518.75 mm^2$.

$A_{pw} = 4700 - 1518.75$

$= 3181.25 mm^2$

$\Rightarrow \frac{3181.25(1600)}{300(1600)(40)} = 0.26.$

0.25 – 1 Interpolation = 1.0

0.30 – 1

$\frac{f_{pu}}{0.87 f_p} = 1$

$\Rightarrow f_{pu} = (1)(0.87)(1600) = 1392 N/mm^2$

$\frac{x_u}{d} = 0.265.$

0.25 – 0.542 → ①

0.26 – x → ②

0.30 – 0.655 → ③

(2) − (1) $\quad 1.1*10 \quad 0.0271$

$8.25x -$

(2)-(1) $0.01 = x - (0.542)$

(3)-(1) $0.05 = 0.113$

$$x = \frac{1.1*10^{-3} + 0.027}{0.05}$$

$$\boxed{x = 0.56}$$

$0.26 = 0.56$

$$\frac{x_u}{d} = 0.56$$

$x_u = 0.56(1600) = 896\text{mm}.$

$D_f = 150\text{mm}.$

Where

$x_u > D_f$.

$M_U = f_{pu} A_{pw} [d - 0.42 x_u] + 0.45 f_{ck} [b - b_w] D_f [d - 0.5 D_f]$

$= (1392)(3181.25) [1600 - 0.42(896)] + 0.45(40) [1200 - 300]$

$150 [1600 - 0.5(150)]$

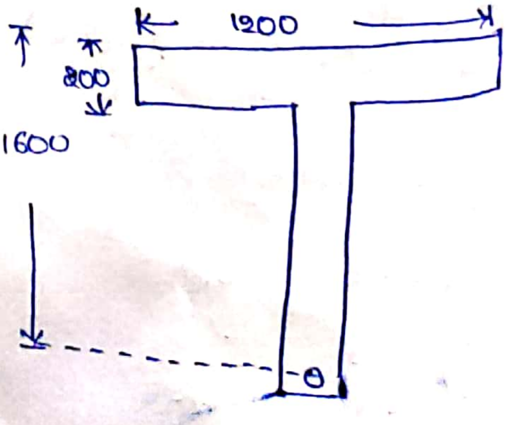
$= 4428300 (1223.68) + 16200 (228750)$

$M_U = 9124 *10^{6}$ kN-m

$\boxed{M_U = 9124.5 \text{ kN-m}}$

7/9/2020.

3) A post tensioned prestressed concrete T-beam having a flange width of 1200mm and thickness of flange 200mm, thickness of web deep be 300mm is prestressed by 2000mm² of high tensile steel located at an effective depth of 1600mm. If $f_{ck} = 40 N/mm^2$, $f_p = 1600 N/mm^2$, estimate the ultimate flexural strength of the unbonded T-section - Assuming span to depth ratio as 20 and $f_{pe} = 1000 N/mm^2$



Sol: Given data.

$b_f = 1200\text{mm}$

$d_f = 200\text{mm}.$

$t_w = 300\text{mm}$

$A_p = 2000\text{mm}^2$

effective depth(d) = 1600mm.

$f_{ck} = 40 N/mm^2$

$f_{pe} = 1000\ N/mm^2$

$\frac{l}{d} = 20.$

Assumming the Neutral axis to fall within the flange. We have the ratio of

$$= \frac{A_p f_{pe}}{b d f_{ck}}$$

$$= \frac{2000(1000)}{1200 * 1600 * 40}$$

$$= 0.026$$

0.025 – 1.34
0.026 – x
0.05 – 1.32

consider 1.34.

$$\Rightarrow \frac{f_{pu}}{f_{pe}} = 1.34$$

$$f_{pu} = 1.34[1000] = \underline{1340}\ N/mm^2.$$

$$\Rightarrow \frac{x_u}{d} = 0.10$$

$$x_u = 0.10(1600)$$

$$\boxed{x_u = 160mm}$$

$x_u < d_f$.

consider

$$m = f_{pu} A_p [d - 0.42 x_u]$$

$$= 1340(2000)[1600 - 0.42(160)]$$

$$= 2680000(1532.8)$$

$$\boxed{m = 4107\ kN\text{-}m}$$

4) A Post-tensioned bride girder with unbonded tendons is of box section of overall dimension 1200mm wide* 1800mm deep with wall thickness of 150mm and high tensile steel has an area of 4000mm^2 and is located at an effective depth of 1600mm. The effective Prestress in steel after all losses is 1000N/mm^2 (f_{pe}), effective span of girder is 24m. If $f_{ck} = 40N/mm^2$ and $f_{pu} = 1600N/mm^2$. Estimate the ultimate flexural strength of section. Assume $b_w = 300mm$.

Sol: Given data:

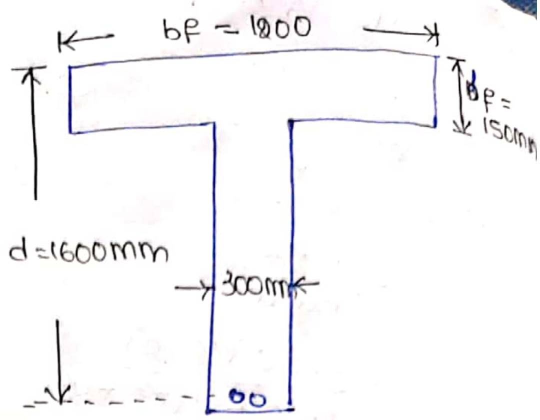
$d_f = 150mm$

$A_p = 4000mm^2$

effective depth $(d) = 1600mm$

$f_{pe} = 1000N/mm^2$

span length = 24m



$b_f = 1200mm$; $D_f = 150mm$

$d_f = 1800mm$.

$b_w = 300mm$.

$f_{pu} = 1600\ N/mm^2$

$f_{ck} = 40\ N/mm^2$.

Effective reinforcement ratio $= \frac{A_{pw} f_p}{b d f_{ck}}$.

$A_{pw} = A_p - A_{pf}$

$A_{pf} = 0.45 f_{ck} [b_f - b_w] \left[\frac{D_f}{f_p}\right]$

$= 0.45\ (40) [1200 - 300] \left[\frac{150}{1600}\right]$

$= 1518.75\ mm^2$

$A_{pw} = A_p - A_{pf}$

$= 4000 - 1518.75$

$= 2481.25\ mm^2$

$\frac{2481.25 (1600)}{300 (1600)(40)} = 0.129$

$\frac{l}{d} = \frac{24 \times 10^3}{1600} = 15$ Consider $\frac{l}{d} = 10$.

0.10 — 1.45 → ①

0.129 — x → ②

0.15 — 1.36 → ③

(2) − (1) ⇒ $0.029 = x - 1.45$

(3) − (1) → $0.05 = -0.09$

$-\cancel{0.0018} = 0.05x - 0.0725$

-2.61×10^{-3}

$D_f = 150mm$

$\boxed{x = 1.38}$

$$\frac{f_{pu}}{f_{pe}} = 1.38$$

$$f_{pu} = 1.38(1000)$$
$$= \underline{1380}\ N/mm^2$$

$$\frac{x_u}{d} = 0.129.$$

0.10 — 0.36

0.129 — x

0.15 — 0.52.

$$0.52 + \frac{[0.52-0.36]}{0.15-0.10} * (0.129 - 0.15) \qquad 0.021$$

$= 0.45$

$$\frac{x_u}{d} = 0.45$$
$$x_u = 0.45(1600)$$
$$= 720\ mm$$
$$x_u > D_f.$$

$$M_u = f_{pu}A_{pw}[d - 0.42x_u] + 0.45f_{ck}[b-b_w]D_f[d-0.5D_f]$$

$$= (1380)(2481.25)[1600 - 0.42(720)] + 0.45(40)[1200-300]150[1600 - 0.5(150)]$$

$$= 4443.4 \times 10^6 + 3705.75 \times 10^6$$

$$\boxed{M_u = 8148\ kN\text{-}m}$$

10/2/2020

Main reasons for controlling deflection:

Large Deflections:

→ Under dynamic effect and unr

→ Under variable loading.

Excessive Deflections:

→ Damage to finishers

→ Partitions

→ Associated structures.

** factors influencing deflection :

1) Imposed load and self weight.
2) magnitude of Prestressing force.
3) span of member.
4) Cable profile.
5) young's modolous of concrete.
6) moment of Inertia (or) 2nd moment of area of cross-section.
7) shrinkage, creep and relaxation of stress
8) Fixidity conditions.

Calculation of deflections:

These are two types.

1. Post cracking
   → It is similar R.C.C
   R.C.C is " to P.S.C.
   → moment curvature

2. Pre cracking.
   → mom whole section is conside moment using mohr's 2nd theorem.

Deflection calculation: 2nd moment area of c/s sectio (or) 2nd moment Inertia.

Mohr's first law:

slope = $\frac{\text{Area of BMD}}{\text{Flexural rigidity}}$.

$$\theta = \frac{A}{EI}$$

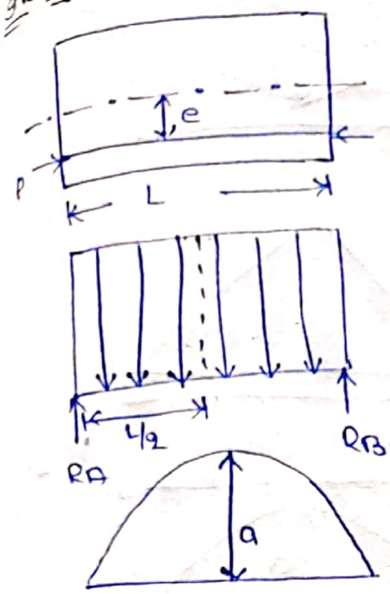
mohr's second law:

$a = \frac{\text{moment of area of BMD}}{\text{flexural rigidity}}$

$$= \frac{A\bar{x}}{EI}.$$

Effect of tendon profile on deflection:-

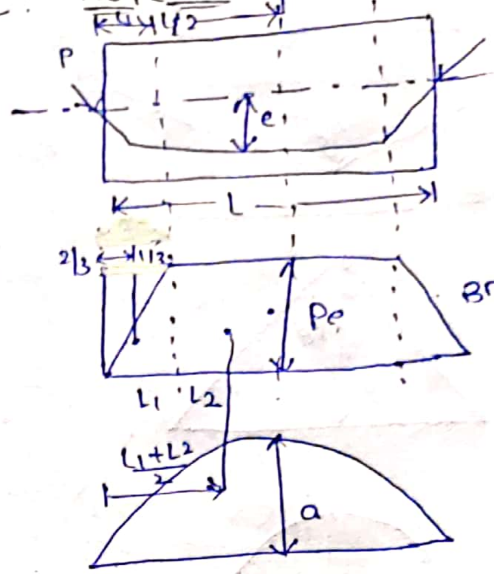
Straight tendon:



$$a = \frac{PeL^2}{8EI} \text{ (upward).}$$

11/9/2020

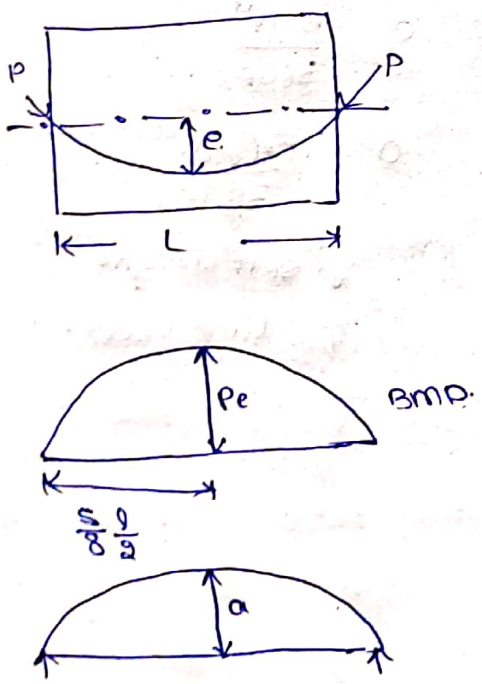
Trapezoidal Tendon:



$$a = \frac{-Pe}{6EI}\left[2l_1^2 + 6l_1l_2 + 3l_2^2\right]$$

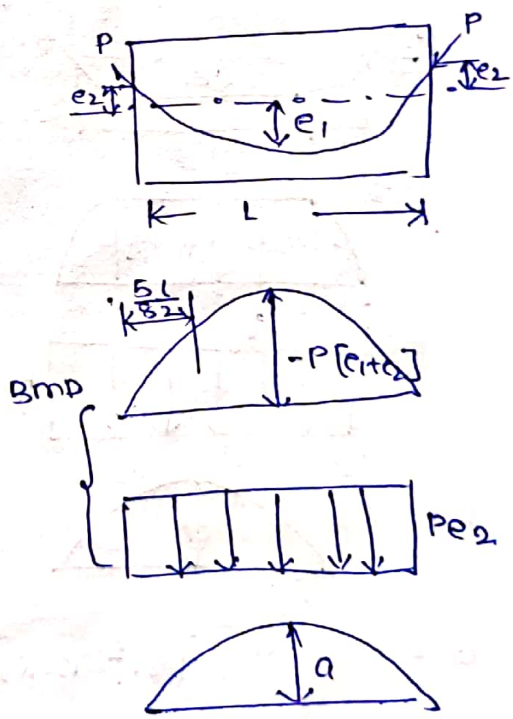
3. Parabolic tendon

1) center



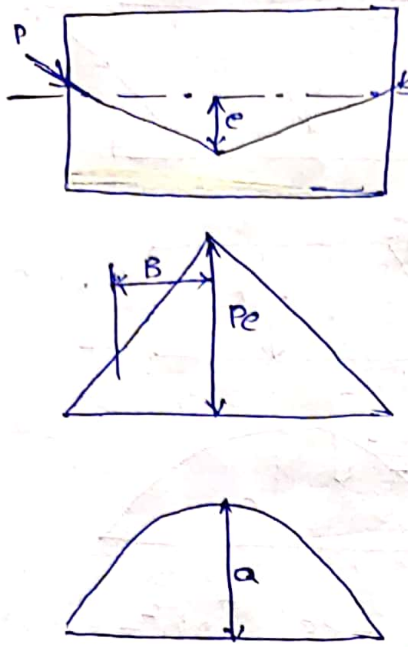
$$a = \frac{-5Pel^2}{48EI}.$$

2) eccentric



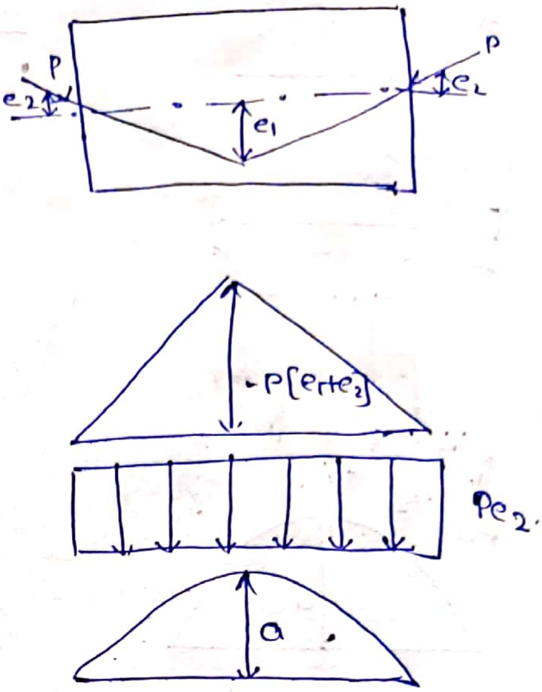
$$a = -\frac{Pl^2}{48EI}\left[-5e_1+e_2\right]$$

Triangular Tendon:



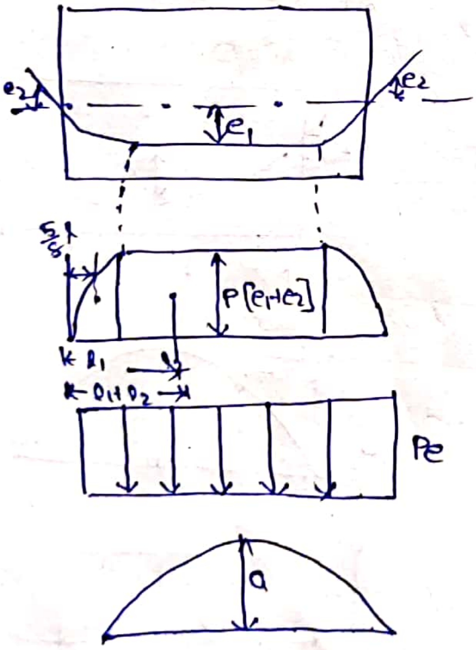
$$a = \frac{Pel^2}{12EI}$$

Slopping tendon



$$a = \frac{Pl^2}{24EI}\left[-2e_1+e_2\right].$$

Parabolic straight tendon.



$$a = \frac{-P(e_1+e_2)}{12EI}\left[5l_1^2+12l_1l_2+6l_2^2\right]+\frac{Pe_2l^2}{8EI}.$$

Deflection due to selfweight & imposed load

$$a = \frac{5wl^4}{384EI}$$

$$a = \frac{5(g+q)l^4}{384EI}.$$

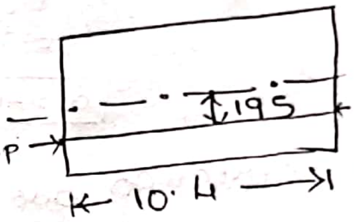
g = self weight

q = live load.

1. The deck of a Prestressed concrete colvertor made up a slab 500mm thick. The slab is spanning over 10.4m and supports a total UDL compressing the dead & live loads of 23.5 kN/m. The modolous of elasticity of concrete (Ec) = 38kN/mm². The concrete slab is prestressed by straight cables each containing 12 high tensile wires of 7mm dia. is stressed to 1200 N/mm² at a constant eccentricity of 195mm. The cables are spaced at 328mm intervals in the transverse direction. Estimate the instantaneous deflections of the slab at the centre of the span under prestressed & imposed loads.

1/3/2020

Sol: Given data,



Thickness of slab = 500 mm

stress = 1200 N/mm²

Span length (l) = 10.4m.

Load (dead + live) (g+q) = 23.5 kN/m.

E_c = 38 kN/mm²

No. of wires = 12 ~~mm~~ wires - 7mm ϕ.

eccentricity (e) = 195mm.

Spacing of cable in transverse direction = 328 mm.

Assume width (b) = 1000 mm.

Force in each cable (F) = stress × Area

Area (A) = $n\frac{\pi}{4}(d)^2$

$= 12\ \frac{\pi}{4}(7)^2 = 461.81$

F = ~~88~~ 461.81 × 12

$\boxed{P = 554.17\ kN}$

Hence the prestressing force per meter width of slab is compound as

$$P = \frac{1000 \times 554.17}{328}$$

$$\boxed{P = 1689.54 \text{ KN}}$$

Deflection due to prestressing force (straight cable)

$$a = -\frac{Pel^2}{8EI}$$

$$I = \frac{bd^3}{12} = \frac{(1000)(500)^3}{12} = 10416.66 \times 10^6 \text{ mm}^4$$

$$a = -\frac{(1689.54)(195)(10.4 \times 10^3)^2}{8 \times 38 \times 10416.66 \times 10^6}$$

$= \cancel{9.00} \text{mm}$

$\boxed{a = -11.25\text{mm}}$ ↑ (-ve)

Deflection due to selfweight and deadload

$$a = \frac{5(q+g)l^4}{384EI}$$

$$= \frac{5(23.5 \times 10^3)(10.4 \times 10^3)^4}{384 \times 38 \times 10416.66 \times 10^6}$$

$\boxed{a = 9.043\text{mm}}$ (due to loads +ve).

Resultant deflection $= 9.043 - 11.25$

$\boxed{a = -2.20 \text{ mm}}$ ↑

2) A PSC beam of rectangular beam 120mm wide & 300mm deep span over 6m the beam is prestressed by a straight cable. carrying an effective force of 200kN at an eccentricity of 50mm. $E_c = 38 \text{ kN/mm}^2$ compute the deflection at centre of span for following cases.

i) Deflection under prestress + selfweight.

ii) Find the magnitude of UDL live load which will nullify the deflection due to prestress & selfweight.

Sol: Given data,

$b = 120mm$

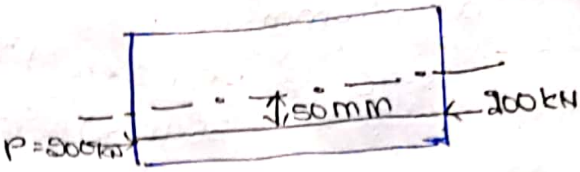
$d = 300mm$.

length $(l) = 6m$.

force $(P) = 200kN$.

$e = 50mm$

$E_c = 38 kN/mm^2$



$$\text{stress} = \frac{\text{force}}{\text{Area}} = \frac{200*10^3}{120*300}.$$

$$\text{stress}(F) = 5N/mm^2.$$

Deflection due to Prestressing force [st. cable]

i)
$$a = \frac{-Pel^2}{8EI}$$

$$I = \frac{bd^3}{12} = \frac{(120)(300)^3}{12} = 270*10^6 mm^4$$

$$a = -\frac{(200)(50)(6*10^3)^2}{8*38*270*10^6}.$$

$$\boxed{a = -4.38 mm}$$

Deflection due to Selfweight.

$$a = \frac{5wl^4}{384EI}.$$

$$w = 0.12*0.3*24 = 0.86 \; kN/m.$$

$$a = \frac{5(0.86*10^{-3})(6*10^3)^4}{384(38)(270*10^6)}$$

$$\boxed{a = 1.42mm}$$

Resultant deflection $= 1.42 - 4.38$

12/12/2020

$$\boxed{a = -2.96mm}$$

ii) Deflection due to live load.

$$a = \frac{5wl^4}{384EI} \qquad w = 9$$

$$a = \frac{59l^4}{384EI}$$

'a' can be taken as +ve; and resultant deflection:

$a = 2.96mm$

$$2.96 = \frac{5(q)(6*10^3)^4}{384(38)(270*10^6)}$$

$1m = 1000mm$

$\frac{1}{1000} m = 1mm$

$kN/m \leftarrow N/m$

$$\boxed{q = 1.8 \, N/mm}$$

$$\boxed{q = 1.8 \, kN/m.}$$

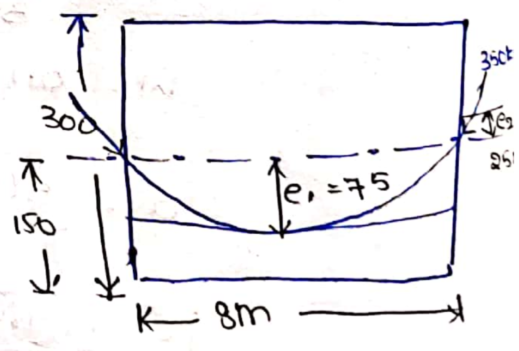
3) A rectangular beam of c/s section 150mm & 300mm deep is simply supported over a span of 8m & is prestressed by means of a symmetric parabolic cable at a distance of 75mm from the bottom of the beam, at mid span & 125mm from the top of the beam at the support sections. If the force in the cable is 350kN and $f_c = 38 \, kN/mm^2$

Calculate :(a) the deflection at mid span when the beam is supporting its own weight.

(b) The concentrated load which must be applied at midspan to restore it to the level of suppo[rts]

Sol: Given data,

$(e_2 = 150 - 125 = 25)$



$E_c = 38 \, kN/mm^2$

$P = 350 \, kN$

$c/s = 150 * 300mm$

$L = 8m$

$e_1 = 75mm$

$e_2 = 25mm.$

a) Deflection due to prestressing force [Parabolic eccentricity tend[on]

$$a = \frac{+PL^2}{48EI}[-5e_1 + e_2]$$

$$I = \frac{bd^3}{12} = \frac{150*300^3}{12} \Rightarrow \boxed{I = 337.5*10^6 \, mm^4}$$

$$a = + \frac{350(8*10^3)^2}{48(38)(337.5*10^6)}\left[-5(75+1)+25\right]$$

$= 0.036(-350)$

$\boxed{a = -12.6 mm}$ ↑

deflection due to self weight

$$a = \frac{5ql^4}{384EI}$$

$q = 24*0.15*0.3 = 1.08$ KN/m.

$q = 1.08$ N/mm.

def

$$a = \frac{5(1.08)(8*10^3)^4}{384(38*10^3)(337.5*10^6)}$$

$\boxed{a = 4.49 mm}$ ↓ (downward)

Prestress at self weight $= 4.49 - 12.6$

$\boxed{a = 8.11 mm}$

(b) concentrated load

$$a = \frac{Wl^3}{48EI} \qquad W = q$$

$$q = \frac{48EIa}{l^3}$$

$$= \frac{48*38*337.5*10^6*8.11}{(8*10^3)^3}$$

$\boxed{q = 9.74 kN}$

1/2/2020

LONG TERM DEFLECTIONS:

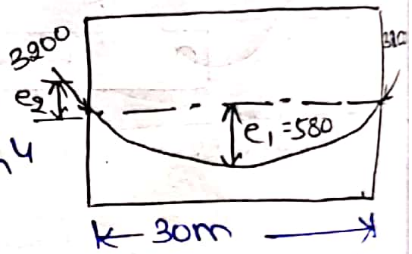
4) A post tensioned roof girder spaning over 30m has an unsymmertical I-section with a 2nd moment area of $72,490*10^6$ mm^4 (I) & an overall depth of 1300mm. The effective eccentricity of the group of parabolic cables at a centre of span is 580mm towards the

Soffit and 170mm towards the top of the beam at Supports. The cables carries an intial Prestressing force of 3200 kN. The Self weight of the girder 10.8 kN/m and the live load of the girdel is 9 kN/m. The modolous of elasicity of concrete is 34 kN/mm². If the creep coefficient is 1.6 and the total loss of prestress is 15%. Estimate the deflections of the following stages & compare them with the permissible limits. values according to IS code: 1343

a) Instaneous deflection due to prestress + self weight

b) Resultant maximum long term deflection allowing for loss of prestress and creep of concrete.

Sol: Given data,



span(l) = 30m.

2nd moment area (I) = 72490×10^6 mm⁴

overall depth (d) = 1300mm

e_1 = 580mm

e_2 = 170mm

Force (P) = 3200 kN.

Self weight of girder (g) = 10.8 kN/m.

live load (q) = 9 kN/m.

E_c = 34 kN/mm².

creep coefficient(ϕ) = 1.6.

Total loss of prestress is 15%.

a) Prestress + self weigth.

$$a = \frac{Pl^2}{48EI}[-5e_1 + e_2]$$

$$= \frac{3200(30 \times 10^3)^2}{48(34)(72490 \times 10^6)}[-5(580) + 170]$$

$$\boxed{a = -66.45 \text{ mm}}$$ ↑(upward)

Deflection due to self weight.

$$a = \frac{5ql^4}{384EI}$$

$$= \frac{5\left(\frac{10.8}{9.1}\right)(30*10^3)^4}{384(3.81)(72490*10^6)}$$

$$\boxed{a = 46.21\ mm}\downarrow$$

Prestress at self weight. = 46.21 − 66.45

$$\boxed{a = -20.23\ mm}$$

212020

Deflection due to live load.

$$a = \frac{5ql^4}{384EI}$$

$$\frac{20.23}{a} = \left[\frac{\cancel{5}\left(\frac{9}{9}\right)(30*10^3)^4}{\cancel{384}(38)(7249*10^6)}\right]\times$$

$$a = \frac{5ql^4}{384EI}$$

$$a = \frac{5(9)(30*10^3)^4}{384*38*7249*10^6}$$

$$\boxed{a = 38.51\ mm}$$

Long term deflection due to creep.

a_i = self weight

a_f = final

$$a_f = [1+\phi]a_i$$

$$a_f = [1+1.6][46.21]$$

$$a_f = 120.14.$$

15% loss of prestress

100 − 15 = 85%

$\frac{85}{100} = 0.85$

= 0.85 * P

= 0.85 * −66.45

= −56.48

Total resultant long term deflection =

38.51 + 120.14 − 56.48

= 108.16 mm

From IS 1343 : Pg: 32 cl: 19.3.1 (or 20.3.1) above value should not be exceed $\frac{span}{250} = \frac{30*10^3}{250}$

= 120 mm

108.16 < 120 mm.

Hence ok.

17/01/2020

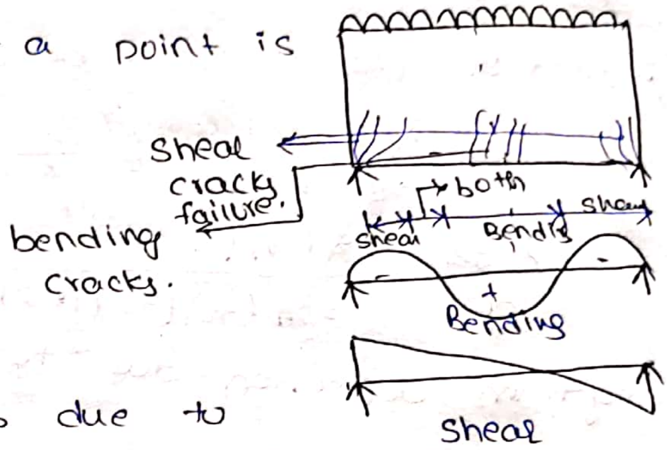
UNIT-5

DESIGN FOR SHEAR & TORSION

Shear and principle stresses:

The shear distribution in an uncracked structural member for which the deformation is assumed to be linear is the function of shear-force and the properties of the cross-section of the member.

The shear stress at a point is expressed as



$$\tau_v = \frac{VA\bar{y}}{Ib} \quad \text{(or)}$$

$$\tau_v = \frac{3}{2}\frac{V_u}{bd}.$$

Where;

τ_v = Shearing stress due to transverse load.

V = Shear force

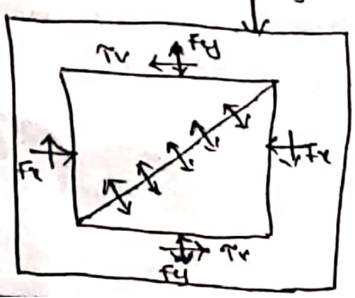
I = moment of Inertia.

b = breadth of member at given point.

In a PSC member the shear stress is generally accompained by a direct stress in the axial direction of the member.

If transverse, vertical prestressing is adopted the compressive stress in the direction perpendicular to the axis of the member will be present in addition to the axial pre-stress. (V_u - $P\sin\theta$)

The most general case of tea an element is subjected to a two-dimensional stress diagram shown in the figure.



Prestress in PSC member.

The maximum and minimum principle stress developed are given by f_{max}, f_{min}.

$$f_{max} = \frac{f_x+f_y}{2} + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}.$$

$$f_{min} = \frac{f_x+f_y}{2} - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

where;

f_x & f_y are direct stresses and τ_v is shear stress acting at a point.

In the PSC member the direct stresses f_x & f_y are compressive the magnitude of the principle tensile stresses is considerbly reduced and in even some cases even eliminated. So that under working loads both major & minor principle stresses are compressive their by eliminating the disc of diagonal tensional cracks.

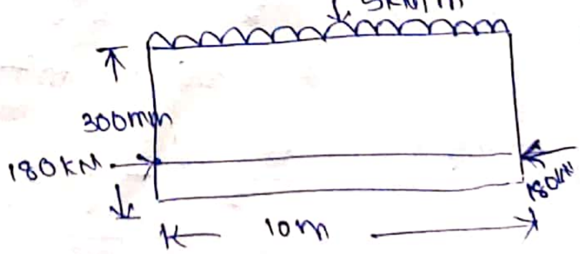
In general there are 3 ways of improving the shear resistance of structural memb concrete member by prestressing technique.

1. Horizontal (or) axial prestressing.
2. Prestressing by inclined (or) sloping.
3. Vertical (or) transverse Prestressing.

8/2/2020

1. A PSC beam of span 10m of rectangular section 120mm wide * 300mm deep is axially prestressed by a cable carrying an effective force of 180KN. The beam supports a total UDL of 5KN/m which includes the selfweight of the member. compare the magnitude of the principle tension developed in the beam with and without axial prestressing.

Sol:



Span (l) = 10m.

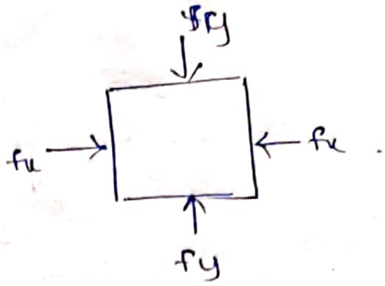
Wide (b) = 120mm

deep (d) = 300mm.

force (F) = 180 KN/m

UDL = 5KN/m.

a) Without axial Prestressing:



$f_x = f_y = 0.$

$\tau_v = \frac{3}{8}\frac{V_u}{bd}.$

$V_u = \frac{wl}{2} = \frac{5(10)}{2} = 25\text{ kN}.$

$\tau_v = \frac{3}{2}\frac{(25\times10^3)}{120\times300}$

$\boxed{\tau_v : 1.04 \text{ N/mm}^2}$

$f_{max}/f_{min} = \frac{f_x+f_y}{2} \pm \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$

Here; $f_x = f_y = 0.$

f_{max} (or) $f_{min} = \pm 1.04 \text{ N/mm}^2$

b) With axial Prestressing

$f_x = \frac{\text{Stress} \times \text{Area}}{\cancel{\text{Area}}}$ & $f_y = 0.$

$\frac{P}{A} = \text{stress}$

$\frac{180\times10^3}{36\times10^3} = \text{stress}$

$\boxed{f_x = 5 \text{ N/mm}^2}$

Principle stress

f_{max}, f_{min}:

$f_{max} = \frac{f_x+f_y}{2} + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{5+0}{2} + \sqrt{\left(\frac{5-0}{2}\right)^2 + (1.041)^2}$

$= 5.2 \text{ N/mm}^2$

$f_{min} = \frac{f_x+f_y}{2} - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{5+0}{2} - \sqrt{\left(\frac{5-0}{2}\right)^2 + (1.041)^2}$

$f_{min} = -0.804 \text{ N/mm}^2$

Comparing magnitude of principal tension.

$\frac{T_v - f_{min}}{T_v} \times 100$

$= \frac{1.041 - 0.2}{1.041} \times 100$

$= 80.78\%$.

2) From the above problem instead of axial prestress-acured cable having an eccentricity 100mm at the centre of the span & redusing to zero at supports is used. The effective force in the cable being 180kN Estimate the percentage redusion in the principal tension in comparsion with the case of axial prestressing.

sol: Given data,

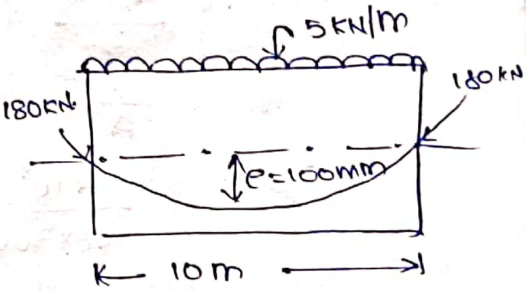
Eccentricity (e) = 100mm.

force (F) = 180 kN.

l = 10m.

wide(b) = 120mm

deep(d) = 300mm.



UDL = 5kN/m.

$V_u - P\sin\theta$.

$\sin\theta$ will be negelible 'θ'

$V_u - P\theta$.

$\theta = \frac{4e}{L}$

$= \frac{4 \times 100}{10 \times 10^3} = 0.04$ rad.

The vertical components of the prestressing force

$= P\sin\theta$.

For smaller values of θ, $\sin\theta$, similar to 'θ' = $P\theta$.

$= 180 \times 10^3 \times 0.04$

$P\theta = 7.2$ kN

Net shear at support $V_u - P\theta$.

$$V_u = \frac{wl}{2}$$

$$= \frac{5(10)}{2} = 25\,kN.$$

$$V_u - P\theta = 25 - 7.2$$

$$\boxed{V_u = 17.8\,kN}$$

maximum shear stress 'τ_v'

$$\tau_v = \frac{3}{2}\frac{V_u}{bd}.$$

$$= \frac{3}{2}\,\frac{17.8 \times 10^3}{120 \times 300}$$

$$\tau_v = 0.741\ N/mm^2$$

The maximum & minimum principal stress.

$$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + (\tau_v)^2}$$

$$f_x = \frac{P}{A}.$$

$$= \frac{180 \times 10^3}{120 \times 300}$$

$$= 5\ N/mm^2$$

$$f_{max} = \frac{5+0}{2} + \sqrt{\left(\frac{5-0}{2}\right)^2 + (0.741)^2}$$

$$= 5.10\ N/mm^2 \text{ (compression)}$$

$$f_{min} = \frac{5+0}{2} - \sqrt{\left(\frac{5-0}{2}\right)^2 + (0.741)^2}$$

$$= -0.107\ N/mm^2 \text{ (tension)}$$

Comparision both fmin

$$\frac{0.207 - 0.107}{0.207} \times 100$$

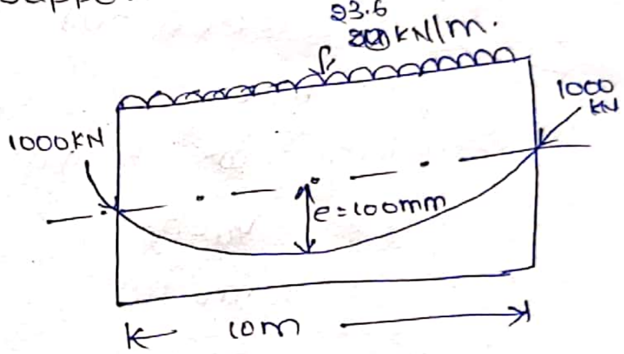
$$= 48\%$$

20/2/2020

3. A concrete beam rectangular c/s section has a width of 250mm & depth of 600mm the wheel is prestressed by a parabolic cable carrying an effective force of 1000KN. The cable is concentric at supports and has maximum eccentricity 100mm at the centre of the span. The beam spans over 10m & supports a UDL liveload 20KN/m (a) Assuming the density of concrete is $24 KN/m^3$, estimate the maximum principal stresses developed in a section of beam at a distance of 300mm from the supports. (b) The prestressing force required to nullify the shear-force due to the dead & live loads at the support section.

Sol:

Given data



P = 1000KN

w = 20 KN/m

e = 100mm.

$\rho = 24 KN/m^3$.

c/s = 250 * 600mm.

l = 10m.

The selfweight of the beam = Area * density of concrete.

= 0.25 * 0.6 * 24

= 3.6 KN/m.

Total load on beam (w) = both dead + live load

= 3.6 + 20

= 23.6 KN/m.

Shear-force at support section.

$$V_u = \frac{wl}{2}$$

$$= \frac{23.6(10)}{2}$$

$$\boxed{V_u = 118 KN}$$

Shear force at a section of 300mm from supports

$V_{at\,300} = V_u - dw$

$= 118 - (0.3)(23.6)$

$V_{at\,300} = 110.92$ KN

$\theta = \frac{4e}{l} = \frac{4(100)}{10 \times 10^3}$

$\theta = 0.04$ rad

Vertical Component of the Prestressing force

$P\sin\theta \simeq P\theta$

$= 1000(0.04)$

$= 40$ KN.

Net shear force at 300mm from the support =

$= V_u - P\theta$

$= 110.92 - 40$

$\boxed{V_u = 70.92 \text{ KN}}$

a) The maximum shear stress at a distance of 300mm at the supports.

$\tau_v = \frac{3}{2} \frac{V_u}{bd}$

$= \frac{3}{2} \frac{[70.92 \times 10^3]}{250 \times 600}$

$\boxed{\tau_v = 0.70 \text{ N/mm}^2}$

The direct Prestressing force

$f_x = \frac{P}{A} = \frac{1000 \times 10^3}{250 \times 600}$ $(\because f_y = 0)$

$= 6.66$ N/mm^2

$f_{max} = \frac{f_x + f_y}{2} + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$

$= \frac{6.66 + 0}{2} + \sqrt{\left(\frac{6.66 - 0}{2}\right)^2 + (0.70)^2}$

$= 6.73$ N/mm^2

$$f_{min} = \left(\frac{f_x+f_y}{2}\right) - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau v^2}$$

$$= \left(\frac{6.66+0}{2}\right) - \sqrt{\left(\frac{6.66-0}{2}\right)^2 + (0.70)^2}$$

$$= -0.79\ N/mm^2$$

b) If prestressing force required to nullify the shear force at the ~~face~~ phase of the support due to dead and live load.

$V\sin\theta = \theta.$

$V_u - P\sin\theta = 0$

$V_u = P\sin\theta$

$= 1000(0.04)$

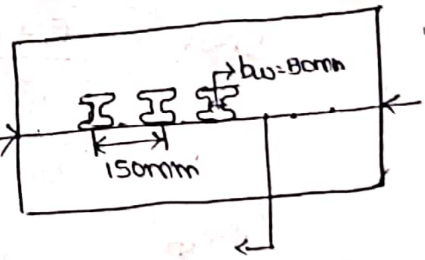
$$\frac{\cancel{70.40}\ 118 \times 10^3}{0.04} = P$$

$$\boxed{P = 2950\,kN}$$

25/2/2020

4. A prestressed I-section has the following properties. Area = 55×10^3 mm²; second moment of Area(I) = 189×10^7 mm⁴; statical moment about centroid = 468×10^4 mm³ $(A\bar{y})$. Thickness of web = 50mm. It is prestressed horizontally by 24 wires of 5mm diameter. and vertically by 24 wires at 150mm centre, all the wires carry similar a tensile stress of 900 N/mm². calculate the principle stresses at the centroid when the shearing force of 80kN acts upon this section.

Sol: Given data;



Area(A) = 55×10^3 mm².

Second moment of Area(I) = 189×10^7 mm⁴.

$(A\bar{y}) = 468 \times 10^4$ mm³

Thickness of web = 50 mm.

horizontal: 24 wires of 5mm φ

vertical = 24 wires of 5mm φ at 150mm centre.

tensile stress = 900 N/mm².

shearing force = 80kN.

$$\tau_v = \frac{VA\bar{y}}{Ib_w A\bar{y}}$$

$$= \frac{80\times10^3 (468\times10^{4})}{189\times10^7 (50) (468\times10^4)}$$

$$\boxed{\tau_v = 3.96 \ N/mm^2}$$

Horizontal Prestress at centre:

$$f_x = \frac{P}{A}$$

P = stress × Area

$= 900 \times 24 \cdot \frac{\pi}{4}(5)^2$

$= 424.KN.$

$= 424.11 \ KN$

$$f_x = \frac{424.11\times10^3}{55\times10^3} = 7.71 \ N/mm^2$$

$$\boxed{f_x = 7.71 \ N/mm^2}$$

$$f_y = \frac{P}{A} \Rightarrow P = \text{stress} \times \text{Area}$$

$$= 900 \times 150 \times 50 \ \frac{\pi}{4}(5)^2$$

$$= 6758 \ KN. = 17.67\times10^3 \ N.$$

$$f_y = \frac{17.67\times10^3}{150\times 50}$$

$$\boxed{f_y = 2.356 \ N/mm^2}$$

maximum principal stress:

$$f_{max} = \left(\frac{f_x+f_y}{2}\right) + \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{7.71+2.35}{2}\right) + \sqrt{\left(\frac{7.71-2.35}{2}\right)^2 + (3.96)^2}$$

$$\boxed{f_{max} = 9.81 \ N/mm^2}$$

minimum principal stress:

$$f_{min} = \left(\frac{f_x+f_y}{2}\right) - \sqrt{\left(\frac{f_x-f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{7.71+2.35}{2}\right) - \sqrt{\left(\frac{7.71-2.35}{2}\right)^2 + (3.96)^2}$$

$$\boxed{f_{min} = 0.85 \ N/mm^2}$$

5) A cantilever portion of a prestressed concrete bridge has a rectangular cross-section 600mm wide & 1650mm deep is 8m long. carries a reaction of 350 kN from the suspended span at free end together with a UDL of 60 kN/m inclusive of its own weight. The beam is prestressed by 7 cables each carrying a force of 1000 kN of which 3 are located at 150mm, 3 at 400mm & 1 at 750mm from the top edge. calculate the magnitude of the principle stresses at a point 550mm from the top cantilever at the support section.

4/2/2020

Sol: Given data.

c/s section of beam = 600mm × 1650mm.

long(l) = 8m.

Point load = 350 kN at freeend.

UDL = 60 kN/m.

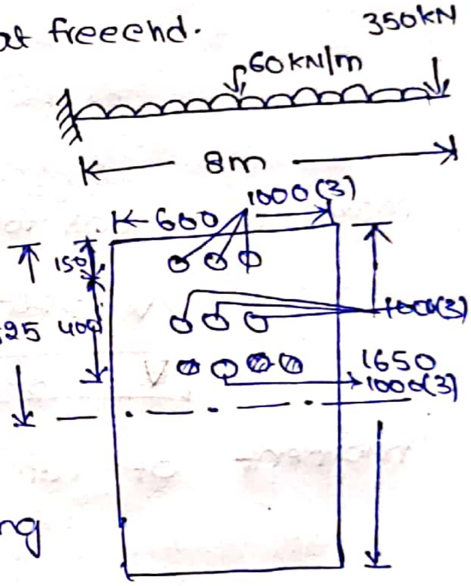
No of cables = 7.

3 at 400mm;

3 at 150mm;

1 at 750mm;

Force (F) = 1000 kN.



4/3/2020

centroid of the prestressing force from the top edge.

$\bar{y} = A\bar{x}$

$y_1 = A_1 x_1 = 3000(150)$

$y_2 = A_2 x_2 = 3000(400)$

$y_3 = A_3 x_3 = 1000(750)$.

$$\bar{y} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{3000(150) + 3000(400) + 1000(750)}{3000 + 3000 + 1000}$$

$$\boxed{\bar{y} = 342.85\text{mm}}$$

Eccentricity (e) = $\frac{d}{2} - \bar{y}$

$= \frac{1650}{2} - 342.85$

825

$\boxed{e = 482.15 mm}$

Total Prestressing force (P) = 3000+3000+1000

P = 7000 KN.

moment due to prestressing force = load * llar distance

$m = P * e.$

$= 7000 * 482.15$

$m = 3.37 * 10^6$ KN-mm

$\boxed{m = 3.37 * 10^6 \text{ KN-m}}$

$\boxed{m = 3375.05 \text{ KN-m}}$

maximum shear force of cantilever beam

PV 350+60(8)

$\boxed{V = 830 \text{ KN}}$

moment of Cantilever beam (m) = moment * distance

$= \cancel{3375.05} * \frac{8}{2}$

$= 350(8) * \frac{8}{2}$

$\boxed{m = 4720 \text{ KN-m}}$

Moment due to live load + dead load is 4720 KN-m

$f_x = \frac{P}{A} + \frac{Pey}{I} - \frac{my}{I}$ (max. resultant direct stress at 550mm from the top edge of support section).

$I = \frac{bd^3}{12} = \frac{600(1650)^3}{12}$

$\boxed{I = 224.60 * 10^9 mm^4}$

$f_x = \frac{830}{160}$

$y = 825 - 550$

$= \frac{1650}{2}$

$= 825$

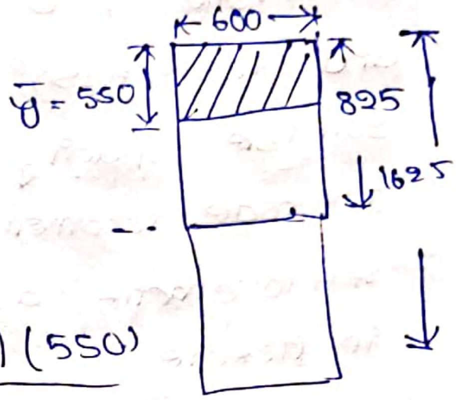
$\boxed{y = 275 mm}$

$$F_x = \frac{7000 \times 10^3}{600(1650)} + \frac{337.8 \times 10^6 \times 10^3 \times 275}{224.60 \times 10^9} - \frac{4720 \times 10^6 (275)}{224.60 \times 10^9}$$

$$= 7.07 + 4.12 - 5.4$$

$$\boxed{F_x = 5.79 \ N/mm^2}$$

The maximum shear stress of 550mm from the top of the flange.



$$\tau_v = \frac{3}{2} \frac{V A \bar{y}}{Jb}$$

$$A = 550(600)$$

$$A = 330 \times 10^3 \ mm^2$$

$$\tau_v = \frac{(830 \times 10^3)(550 \times 600)(550)}{224.60 \times 10^9 \times 600}$$

$$\boxed{\tau_v = 1.11 \ N/mm^2}$$

$\therefore f_y = 0$

maximum principal stress

$$f_{max} = \left(\frac{f_x + f_y}{2}\right) + \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{5.79 + 0}{2}\right) + \sqrt{\left(\frac{5.79 - 0}{2}\right)^2 + (1.11)^2}$$

$f_{max} = 5.99 \ N/mm^2 \simeq \underline{6 \ N/mm^2}$ (compression)

minimum principal stress

$$f_{min} = \left(\frac{f_x + f_y}{2}\right) - \sqrt{\left(\frac{f_x - f_y}{2}\right)^2 + \tau_v^2}$$

$$= \left(\frac{5.79 + 0}{2}\right) - \sqrt{\left(\frac{5.79 - 0}{2}\right)^2 + (1.11)^2}$$

$$\boxed{f_{min} = -0.205 \ N/mm^2}$$ (Tension)

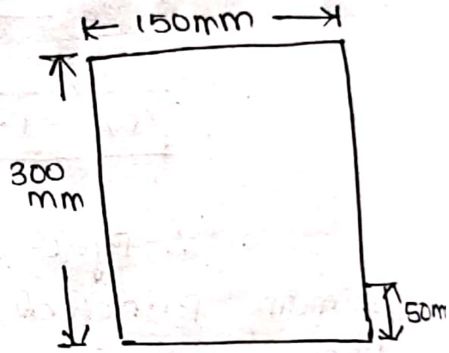
SHEAR

1. A prestress member of ▭lar section girder of 150mm wide & 300mm deep to be designed to support and ultimate shear force of 130 KN. The uniform prestress across the section is 5 N/mm² given the characteristic cube strength of concrete as 40 N/mm² and Fe415 HYSD bars of 8mm diameter. Design suitable spacing for stirrup confirming to the I.S code recommendations. Assume cover to the reinforcement as 50mm & the section uncracked in flexure.

Sol:

B = 150mm

D = 300mm.



Shear force (V) = 130KN

~~ultimate~~ shear

uniform prestress (f_p) = 5 N/mm²

F_{ck} = 40 N/mm²

Fe415 HYSD bars of 8mm ϕ

effective cover = 50mm.

Uncracked section in flexure:

$V_c = V_{co}$ ∴ Pg No. 32 23.4.1

$V_{co} = 0.67\, b D \sqrt{f_t^2 + 0.8 f_{cp} f_t}$ Cl No: 22.4.1

$f_t = 0.24\sqrt{f_{ck}}$.

$= 0.24\sqrt{40}$

$\boxed{f_t = 1.51\ N/mm^2}$

$f_{cp} = 5\ N/mm^2$

$$V_{CO} = 0.67 * 150 * 300\sqrt{(1.51)^2 + (0.8)(5)(1.51)}$$

$$\boxed{V_{CO} = 86.96\ kN}$$

$V > V_{CO}$ (or) V_c.

shear force (130) > 86.96.

∴ provide shear reinforcement as per cl No. 23.4.3.2

pg: 33

$$\boxed{\frac{A_{sv}}{S_v} = \frac{V - V_c}{0.87 f_y d_t}}$$

Assume 2legged 8mm ϕ vertical stirrups.

$A_{sv} = 2 \cdot \frac{\pi}{4}(8)^2 = 100.53\ mm^2$.

$$\frac{100.53}{S_v} = \frac{130 - 86.96}{0.87 * 415 * 250}$$

$d_t = d - cover$.

$= 300 - 50$

$= 250\ mm$.

$$S_v = \frac{100.53 * 0.87 * 415 * 250}{130 * 10^3 - 86.96 * 10^3}$$

$$\boxed{S_v = 210.82\ mm}$$

$S_v \not> 0.75\ d_t$

22.4.3.2

$= 0.75(250)$

$= 187.5\ mm$

$S_v \not> 4b_w = 4(150) = 600$

consider less value.

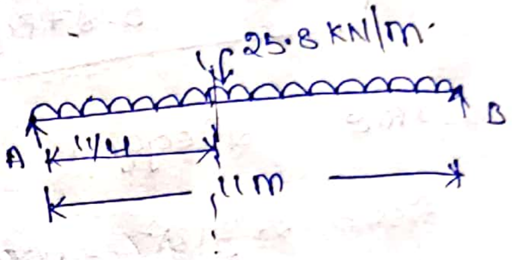
∴ provide 2legged 8mm ϕ at 187.5 mm c/c.

6/3/2020.

3) A Pretensioned beam of rectangular c/s section 250mm x 550mm has an effective pretensi prestressing force of 900KN at an constant eccentricity of 200mm. It carries a total service load of 25.8 kN/m over an effective span of 11m. Design a shear reinforcement for the beam. The grade of concrete is 40. Design a shear reinforcement at support section & at 1/4th of the span.

Sol:

Cracking flexural:



Given data;

B = 250mm

D = 550mm.

force = 900 kN.

e = 200mm.

load (P) = 25.8 kN/m

span (l) = 11m.

grade of concrete = 40.

At 1/4 span:

$\frac{wl}{2} - wx.$

shear force = load * Distance

$RA - wx.$

$= \frac{25.8 \times 11}{2} - \left(\frac{11}{4}\right) 25.8$

$RA = RB = \frac{wl}{2}$

$= \frac{25.8 \times 11}{2} - \left(\frac{11}{4}\right) 25.8$

$V = 70.95 kN$

$RA = \frac{25.8(11)}{2}$

moment (m) $= RA \times x - w.x.\frac{x}{2}$ (or) $\frac{3wl^2}{32}$

$= 11 * (12.9) * \frac{11}{4} - (25.8) \frac{11}{4} * \left(\frac{\frac{11}{4}}{2}\right)$

$= 390.225 - 97.56$

$= 292.66$ kNm

Assume Pre stressing (fp) = 1600 N/mm² in steel.

cracking flexural

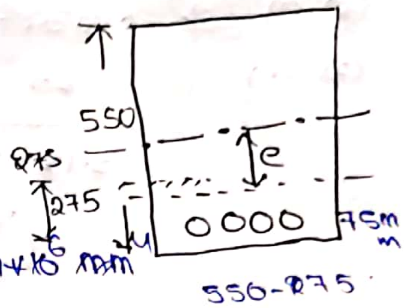
$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_p}\right)\tau_0 b d + M_0 \frac{V}{M}$$

Pg:47

$f_{pe} = 0.6\, f_p$

$= 0.6(1600)$

$= 960\ N/mm^2$.



$\Rightarrow$ $\boxed{M_0 = 0.8\, f_{pt} \cdot \frac{I}{y}}$

$I = \frac{bd^3}{12} = \frac{250(500-75)^3}{12} = 3466.61 \times 10^6\ mm^4$

$I = 2232.74 \times 10^6\ mm^4$

$f_{pt} = \frac{P}{A} + \frac{Pey}{I}$.

$= \frac{900 \times 10^3}{250 \times 550} + \frac{90 \times 10^3 \times 200 \times 200}{2232.74 \times 10^6}$

$y = 275 - 75$
$= 200\,mm$

$f_{pt} = 6.54 + 16.12$

$\boxed{f_{pt} = 22.66\ N/mm^2}$

$M_0 = 0.8\, f_{pt} \cdot \frac{I}{y}$.

$= 0.8(22.66)\frac{(2232.74 \times 10^6)}{200}$

$\boxed{M_0 = 202.37\ kN\text{-}m}$

$\rightarrow$ for τ:

$100\frac{A_p}{bd} = A_{pe}$ ∴ from table 6 : pg.47.

$A_p = \frac{P}{f_{pe}}$.

$= \frac{900 \times 10^3}{960}$

$A_p = 937.5\ mm^2$.

$100\left(\frac{937.5}{250 \times 475}\right) = 0.789$

For M40:

0.75 — 0.6 → (1)

0.789 — τ → (2)

1.00 — 0.68 → (3).

(2)-(1) ⇒ 0.039 — (τ-0.6)

(3)-(1) ⇒ 0.25 — 0.08

$3.12 \times 10^{-3} = 0.25\tau - 0.15$

$\boxed{\tau_c = 0.61\ N/mm^2}$

$V_{cr} = \left[1 - 0.55\frac{f_{pe}}{f_p}\right]\tau_c bd + M_0 \cdot \frac{V}{M}$

$= \left[1 - 0.55\frac{960}{1600}\right](0.61)(250)(475) + 202.3 \times 10^6 \cdot \frac{70.95 \times 10^3}{292.66 \times 10^6}$

= 97.58 KN

9/3/2020

At Corner Support Section:

Shear force (V) = ~~M~~

We have to provide shear reinforcement.

$\frac{A_{sv}}{b s_v} = \frac{0.4}{0.87 f_y}$

Assume A_{sv} = 2 legged 8mm ϕ

$A_{sv} = 2\left(\frac{\pi}{4}\right)(8^2) = 100.53 mm^2$

$\frac{100.53}{250(s_v)} = \frac{0.4}{0.87(415)}$

$\boxed{s_v = 362.96 mm}$

At support section:

shear force $(V) = \frac{wl}{2}$.

$= \frac{25.8(11)}{2}$

$V_e = 141.9$ kN.

(pg: 47)

$V_{cr} = 0.1 bd\sqrt{f_{ck}}$

$= (0.1)(250)(475)\sqrt{40}$

$\boxed{V_{cr} = 75.10 \text{ kN}}$

$\boxed{V > V_{cr} \text{ (or) } V_c}$

We have to provide shear reinforcement — 22.4.3.2.

$$\frac{A_{sv}}{S_v} = \frac{V - V_c}{0.87 f_y d_t}.$$

Assume Asv is 2legged 8-mm ϕ.

$A_{sv} = 2 \cdot \frac{\pi}{4}(8)^2 = 100.53 \text{ mm}^2$

$$\frac{100.53}{S_V} = \frac{(141.9 * 10^3) - (75.10 * 10^3)}{0.87(415)(475)}$$

$$\frac{100.53}{S_V} = 0.38$$

$\boxed{S_v = 264.55 \text{mm}}$

Check:

$S_v \ngtr 0.75 d_t$

$0.75(475)$

356.25.

$S_v \not> 4b_w \Rightarrow 4(250)$

$= 1000.$

consider less value.

2 legged 8mmϕ at 264.55mm. cross-section.

7.1 Transmission of Prestress (Part I)

This section covers the following topics.

- Pre-tensioned Members

7.1.1 Pre-tensioned Members

The stretched tendons transfer the prestress to the concrete leading to a self equilibrating system. The mechanism of the transfer of prestress is different in the pre-tensioned and post-tensioned members. The transfer or transmission of prestress is explained for the two types of members separately.

For a pre-tensioned member, usually there is no anchorage device at the ends. The following photo shows that there is no anchorage device at the ends of the pre-tensioned railway sleepers.

Figure 7-1.1 End of pre-tensioned railway sleepers

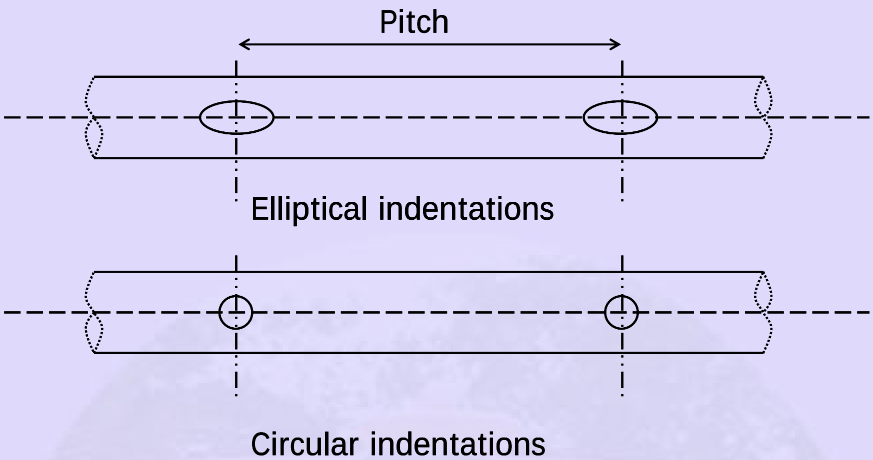
(Courtesy: The Concrete Products and Construction Company, COPCO, Chennai)

For a pre-tensioned member without any anchorage at the ends, the prestress is transferred by the bond between the concrete and the tendons. There are three mechanisms in the bond.

1) Adhesion between concrete and steel
2) Mechanical bond at the concrete and steel interface

3) Friction in presence of transverse compression.

The mechanical bond is the primary mechanism in the bond for indented wires, twisted strands and deformed bars. The surface deformation enhances the bond. Each of the type is illustrated below.



Examples of indented wires

Twisted strand

Deformed bar

Figure 7-1.2 Indented wires, twisted strands and deformed bars

The prestress is transferred over a certain length from each end of a member which is called the **transmission length** or **transfer length** (L_t). The stress in the tendon is zero at the ends of the members. It increases over the transmission length to the effective prestress (f_{pe}) under service loads and remains practically constant beyond it. The following figure shows the variation of prestress in the tendon.

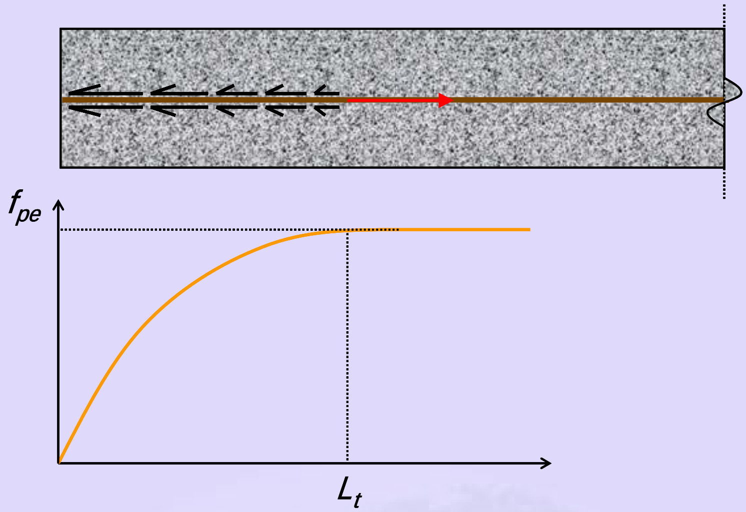


Figure 7-1.3 Variation of prestress in tendon along transmission length

Hoyer Effect

After stretching the tendon, the diameter reduces from the original value due to the Poisson's effect. When the prestress is transferred after the hardening of concrete, the ends of the tendon sink in concrete. The prestress at the ends of the tendon is zero. The diameter of the tendon regains its original value towards the end over the transmission length. The change of diameter from the original value (at the end) to the reduced value (after the transmission length), creates a wedge effect in concrete. This helps in the transfer of prestress from the tendon to the concrete. This is known as the Hoyer effect. The following figure shows the sequence of the development of Hoyer effect.

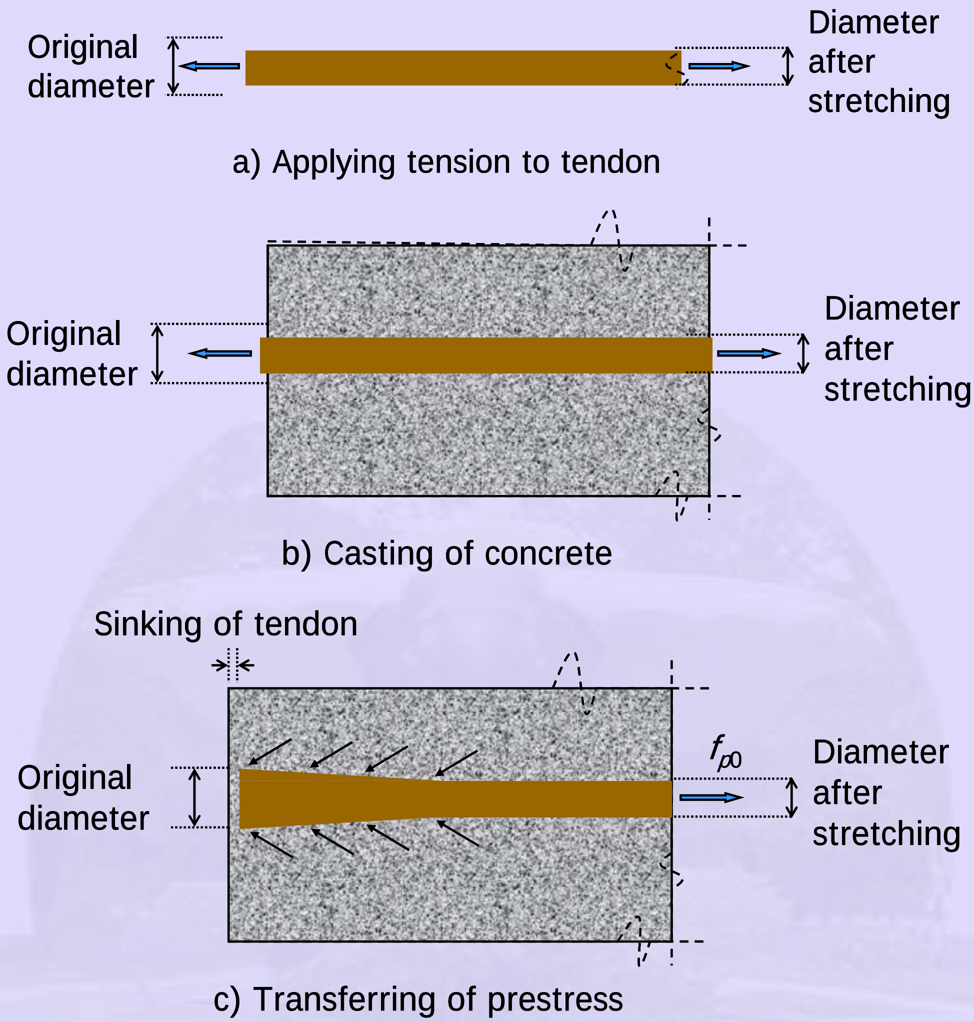


Figure 7-1.4 Hoyer effect

Since there is no anchorage device, the tendon is free of stress at the end. The concrete should be of good quality and adequate compaction for proper transfer of prestress over the transmission length.

Transmission Length

There are several factors that influence the transmission length. These are as follows.

1) Type of tendon
   - wire, strand or bar
2) Size of tendon
3) Stress in tendon
4) Surface deformations of the tendon

- Plain, indented, twisted or deformed

5) Strength of concrete at transfer
6) Pace of cutting of tendons
   - Abrupt flame cutting or slow release of jack
7) Presence of confining reinforcement
8) Effect of creep
9) Compaction of concrete
10) Amount of concrete cover.

The transmission length needs to be calculated to check the adequacy of prestress in the tendon over the length. A section with high moment should be outside the transmission length, so that the tendon attains at least the design effective prestress (f_{pe}) at the section. The shear capacity at the transmission length region has to be based on a reduced effective prestress.

IS:1343 - 1980 recommends values of transmission length in absence of test data. These values are applicable when the concrete is well compacted, its strength is not less than 35 N/mm^2 at transfer and the tendons are released gradually. The recommended values of transmission length are as follows.

Table 7-1.1 Values of transmission length

For plain and intended wires	$L_t = 100\ \Phi$
For crimped wire	$L_t = 65\ \Phi$
For strands	$L_t = 30\ \Phi$

Here, Φ is the nominal diameter of the wire or strand.

To avoid the transmission length in the clear span of a beam, **IS:1343 - 1980** recommends the following.

1) To have an overhang of a simply supported member beyond the support by a distance of at least ½ L_t.

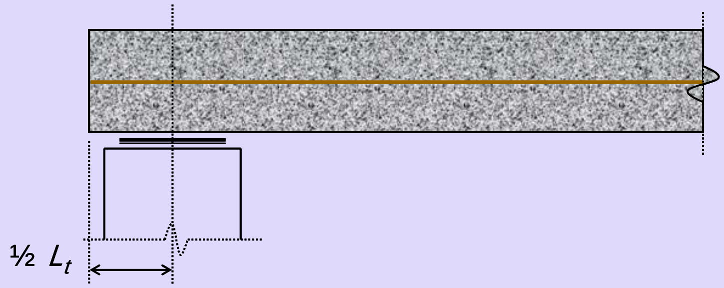


Figure 7-1.5 End of a simply supported member

2) If the ends have fixity, then the length of fixity should be at least L_t.

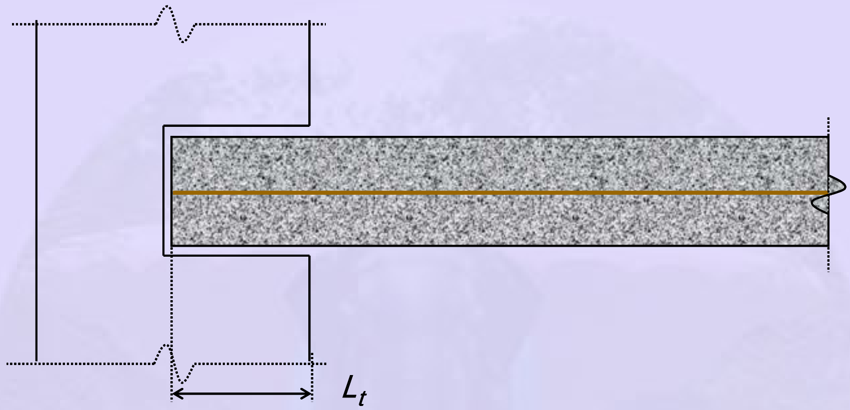


Figure 7-1.6 End of a member with fixity

Development Length

The development length needs to be provided at the critical section, the location of maximum moment. The length is required to develop the ultimate flexural strength of the member. The development length is the minimum length over which the stress in tendon can increase from zero to the ultimate prestress (f_{pu}). The development length is significant to achieve ultimate capacity.

If the bonding of one or more strands does not extend to the end of the member (de-bonded strand), the sections for checking development of ultimate strength may not be limited to the location of maximum moment.

The development length (L_d) is the sum of the transmission length (L_t) and the bond length (L_b).

$$L_d = L_t + L_b \qquad \textbf{(7-1.1)}$$

The bond length is the minimum length over which, the stress in the tendon can increase from the effective prestress (f_{pe}) to the ultimate prestress (f_{pu}) at the critical location.

The following figure shows the variation of prestress in the tendon over the length of a simply supported beam at ultimate capacity.

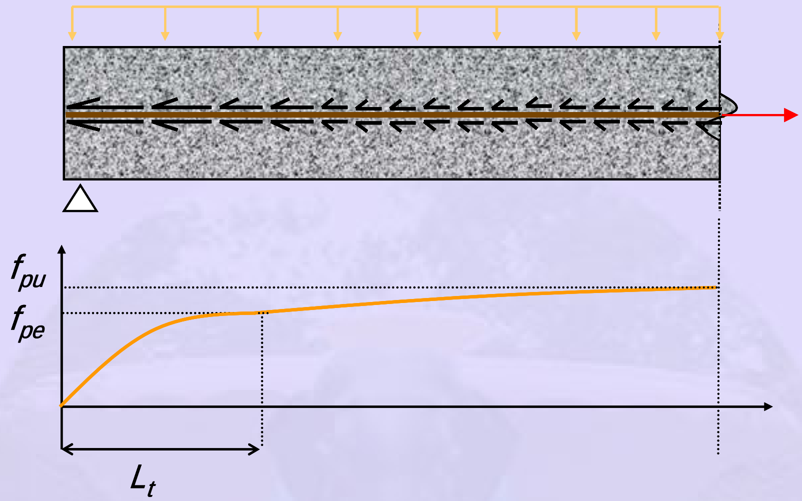


Figure 7-1.7 Variation of prestress in tendon at ultimate

The calculation of the bond length is based on an average design bond stress (τ_{bd}). A linear variation of the prestress in the tendon along the bond length is assumed. The following sketch shows a free body diagram of a tendon along the bond length.

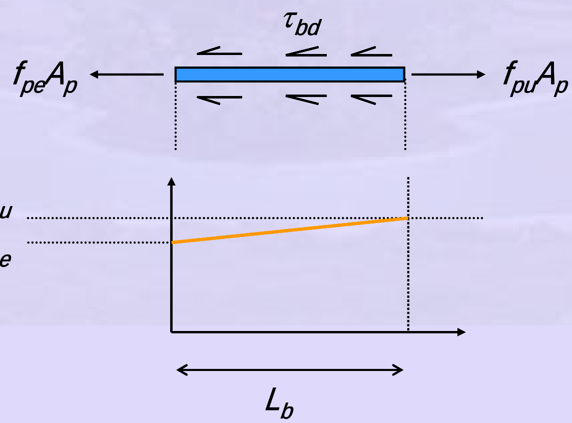


Figure 7-1.8 Assumed variation of prestress in tendon along the bond length

The bond length depends on the following factors.

1) Surface condition of the tendon
2) Size of tendon
3) Stress in tendon

4) Depth of concrete below tendon

From equilibrium of the forces in the above figure, the expression of the bond length is derived.

$$L_b = \frac{\left(f_{pu} - f_{pe}\right)\varphi}{4\tau_{bd}} \qquad \textbf{(7-1.2)}$$

Here, Φ is the nominal diameter of the tendon.

The value of the design bond stress (τ_{bd}) can be obtained from **IS:456 - 2000, Clause 26.2.1.1**. The table is reproduced below.

Table 7-1.2 Design bond stress for plain bars

Grade of concrete	M30	M35	M40 and above
τ_{bd} (N/mm^2)	1.5	1.7	1.9

End Zone Reinforcement

The prestress and the Hoyer effect cause transverse tensile stress (σ_t). This is largest during the transfer of prestress. The following sketch shows the theoretical variation of σ_t.

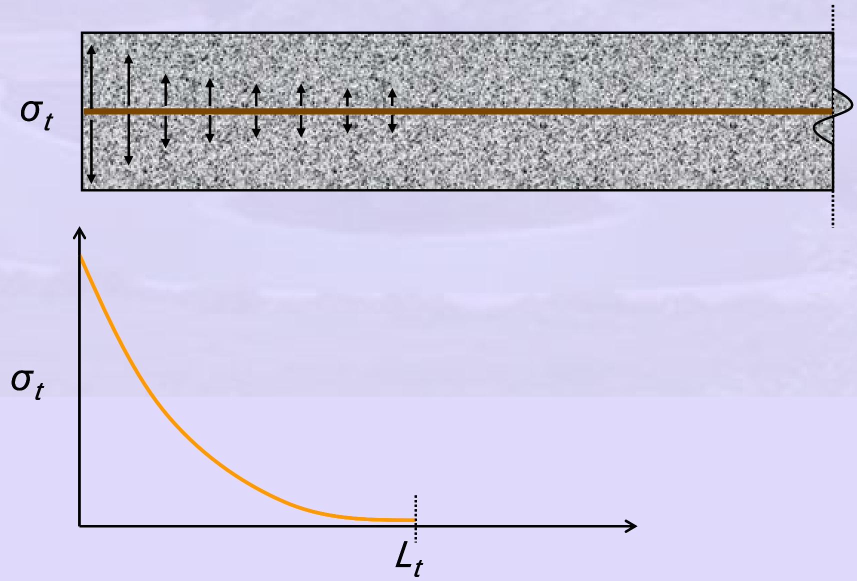


Figure 7-1.9 Transverse stress in the end zone of a pre-tensioned beam

To restrict the splitting of concrete, transverse reinforcement (in addition to the reinforcement for shear) needs to be provided at each end of a member along the transmission length. This reinforcement is known as **end zone reinforcement**.

The generation of the transverse tensile stress can be explained by the free body diagram of the following zone below crack, for a beam with an eccentric tendon. Tension (*T*), compression (*C*) and shear (*V*) are generated due to the moment acting on the horizontal plane at the level of the crack. The internal forces along the horizontal plane are shown in (a) of the following figure. The variation of moment (due to the couple of the normal forces) at horizontal plane along the depth is shown in (b).

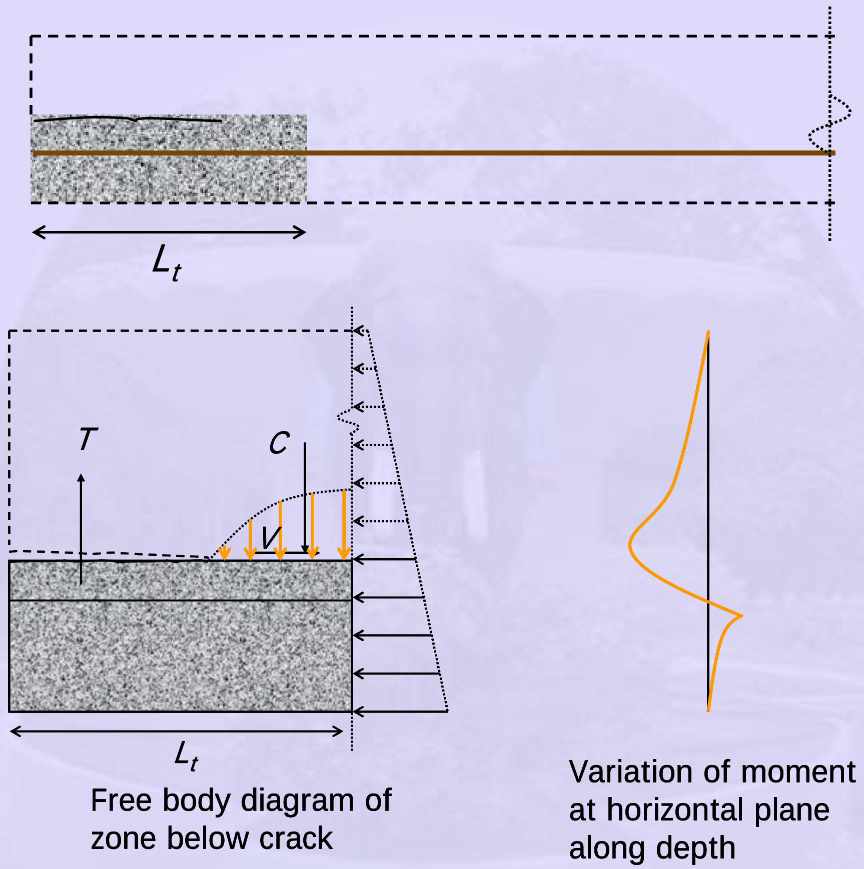


Figure 7-1.10 Forces in the end zone

The end zone reinforcement is provided to carry the tension (*T*) which is generated due to the moment (*M*). The value of *M* is calculated for the horizontal plane at the level of CGC due to the compressive stress block from the normal stresses in a vertical plane above CGC. The minimum amount of end zone reinforcement (A_{st}) is given in terms of the moment (*M*) as follows.

$$A_{st} = \frac{2.5M}{f_s h} \quad \textbf{(7-1.3)}$$

In the previous equation,

h = total depth of the section

M = moment at the horizontal plane at the level of CGC due to the compressive stress block above CGC

f_s = allowable stress in end zone reinforcement.

The lever arm for the internal moment is h/2.5. The value of f_s is selected based on a maximum strain.

The end zone reinforcement should be provided in the form of closed stirrups enclosing all the tendons, to confine the concrete. The first stirrup should be placed as close as possible to the end face, satisfying the cover requirements. About half the reinforcement can be provided within a length equal to $\frac{1}{3}L_t$ from the end. The rest of the reinforcement can be distributed in the remaining $\frac{2}{3}L_t$.

References:

1) Krishnamurthy, D. "*A Method of Determining the Tensile Stresses in the End Zones of Pre-tensioned Beams*", Indian Concrete Journal, Vol. 45, No. 7, July 1971, pp. 286-297.

2) Krishnamurthy, D. "*Design of End Zone Reinforcement to Control Horizontal Cracking in Pre-tensioned Concrete Members at Transfer*", Indian Concrete Journal, Vol. 47, No. 9, September 1973, pp. 346-349.

Example 7-1.1

Design the end zone reinforcement for the pre-tensioned beam shown in the following figure.

The sectional properties of the beam are as follows.

$A = 46,400\ \text{mm}^2$

$I = 8.47 \times 108\ \text{mm}^4$

$Z = 4.23 \times 105\ \text{mm}^3$

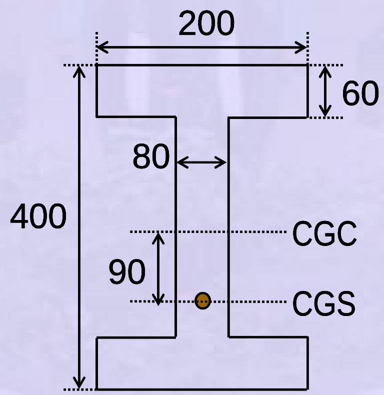
There are 8 prestressing wires of 5 mm diameter.

$A_p = 8 \times 19.6 = 157\ \text{mm}^2$

The initial prestressing is as follows.

$f_{p0} = 1280\ \text{N/mm}^2$.

Limit the stress in end zone reinforcement (f_s) to 140 N/mm^2.



Cross-section at end

Solution

1) Determination of stress block above CGC

Initial prestressing force

$$P_0 = A_p.f_{po}$$

$$= 157 \times 1280\ \text{N}$$

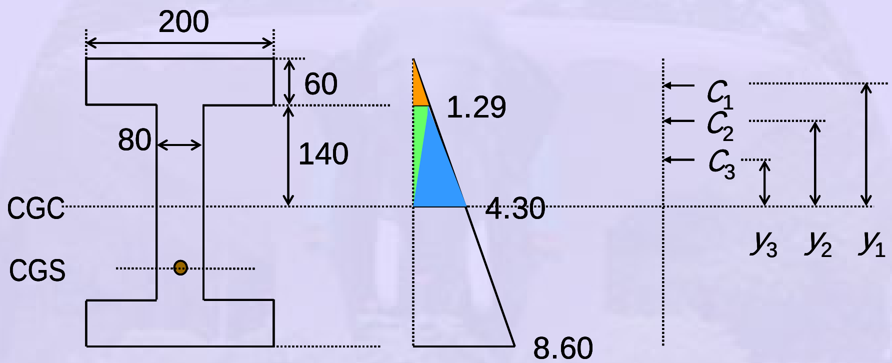
$$= 201\ \text{kN}$$

Stress in concrete at top

$$\begin{aligned} f_t &= -\frac{P_0}{A} + \frac{P_0 e}{Z} \\ &= -\frac{201\times10^3}{46400} + \frac{201\times10^3\times90}{4.23\times10^5} \\ &\approx 0\ \text{N/mm}^2 \end{aligned}$$

Stress at bottom

$$\begin{aligned} f_b &= -\frac{P_0}{A} - \frac{P_0 e}{Z} \\ &= -\frac{201\times10^3}{46400} - \frac{201\times10^3\times90}{4.23\times10^5} \\ &= -8.60\ \text{N/mm}^2 \end{aligned}$$



Stress profile

Components of compression block

2) Determination of components of compression block

C_1 = ½ × 1.29 × 200 × 60

= 7.74 kN

y_1 = 140 + ⅓ × 60

= 160 mm

C_2 = ½ × 1.29 × 140 × 80

= 7.22 kN

y_2 = ⅔ × 140

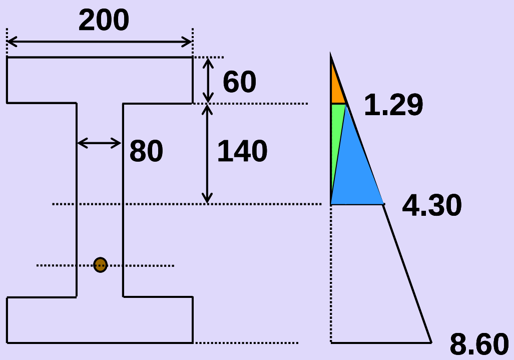
= 93.3 mm

C_3 = ½ × 4.3 × 140 × 80

= 24.08 kN

y_3 = ⅓ × 140

= 46.7 mm



3) Determination of moment

$$M = \Sigma\, C_i . y_i$$
$$= C_1.y_1 + C_2.y_2 + C_3.y_3$$
$$= (7.74 \times 160) + (7.22 \times 93.3) + (24.08 \times 46.7)$$
$$= 3036.6 \text{ kN-mm}$$

4) Determination of amount of end zone reinforcement

$$A_{st} = \frac{2.5M}{f_s h}$$
$$= \frac{2.5\text{M}}{140 \times 400}$$
$$= \frac{2.5 \times 3036.6 \times 10^3}{140 \times 400}$$
$$= 135.6 \text{ mm}^2$$

With 6 mm diameter bars, required number of 2 legged closed stirrups

$$= 135.6 / (2 \times 28.3) \Rightarrow 3.$$

For plain wires, transmission length

$$L_t = 100\, \Phi$$
$$= 500 \text{ mm.}$$

Provide 2 stirrups within distance 250 mm (L_t/2) from the end. The third stirrup is in the next 250 mm.

Designed end zone reinforcement